

ECE 600 Homework 3

1. Let X be a random variable with cdf $F_X(x)$. Show that if $b > a$, then $F_X(b) \geq F_X(a)$.
2. Which of the following are valid cumulative distribution functions?

(a)

$$F_X(x) = \begin{cases} 1 & x > 1 \\ x^2 & 0 \leq x \leq 1 \\ 0 & x < 0 \end{cases}$$

(b)

$$F_X(x) = \begin{cases} 0 & x \leq c \\ 1 & x > c \end{cases}$$

(c)

$$F_X(x) = \begin{cases} 1 & x \geq 3 \\ x - 1 & 2 \leq x < 3 \\ 0 & x < 2 \end{cases}$$

(d)

$$F_X(x) = 1 - e^{-x}$$

(e)

$$F_X(x) = (1 - e^{-x})u(x)$$

where c is a constant and $u(x)$ is the unit step function.

3. When ten fair coins are flipped, the event of interest is the number of heads. Let this number be a random variable. Assuming the ten flips are independent of each other,
 - (a) Plot the probability mass function for this random variable.
 - (b) What is the probability that the random variable is between six and nine inclusive?
 - (c) What is the probability that the random variable is greater than or equal to eight?
4. The median of the random variable X is defined as the real number x_m such that $P(X \leq x_m) = 0.5$. Find the median of the random variable which has the probability density function

$$f_X(x) = \begin{cases} be^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Note: The first step is to find the value of b .

5. Let X be an exponential random variable with parameter λ .
- (a) For $x > 0$, find $P(\{X \leq x\})$.
 - (b) For $x_2 > x_1 > 0$, find $P(\{x_1 \leq X \leq x_2\})$.
 - (c) For $x > 0$, find $P(\{X \geq x\})$.
 - (d) Segment the positive real line into three equiprobable disjoint intervals.
6. The probability density function of X , the lifetime of a certain type of electronic device (measured in hours), is given by

$$f_X(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$$

What is the probability that of 6 such devices at least 3 will function for at least 15 hours? You may assume that the functioning of each device is independent of the functioning of the other devices.