COMER
October 7, 2010

ECE 600 Exam 1

1. Enter your name and signature in the space provided below.
2. You may not use a calculator or any other reference materials.
3. Partial credit will be given, at the discretion of the instructor.

Name:

SOLUTION

Signature:

1. (30 points) Consider an experiment in which a die is rolled repeatedly until a six is rolled, at which point the experiment stops. The outcome of this experiment is the sequence of values rolled. Note: For this problem it is important that your notation for the sample space and the event space are consistent.

   (a) (10 points) Define the sample space for this experiment.

   (b) (10 points) Let E_n be the event that the total number of rolls made is n. What outcomes are in E_n?

   (c) (10 points) What outcomes are in $(\cup_{n=1}^{\infty} E_n)^c$?

(a) $S = \{$ sequences $x_1, x_2, x_3, \ldots, 6 :$ $x_i \in \{1,2,3,4,5\} \; \forall i \}$

(b) $E_n = \{$ sequences $x_1, x_2, \ldots, x_{n-1}, 6 :$ $x_i \in \{1,2,3,4,5\} \; \forall i = 1, \ldots, n-1 \}$

(c) $\bigcup_{n=1}^{\infty} E_n = S$

So $\left(\bigcup_{n=1}^{\infty} E_n\right)^c = \emptyset$

$\Rightarrow$ no outcomes are in $\left(\bigcup_{n=1}^{\infty} E_n\right)^c$

2. (20 points) Let F_1 and F_2 be σ−fields in S. Show that $F_1 \cap F_2$ is also a σ−field.

$F_1 \cap F_2$ is nonempty because $S \in F_1$ and $S \in F_2$, so $S \in F_1 \cap F_2$.

Let $A \in F_1 \cap F_2$. Then $A \in F_1$ and $A \in F_2$. So $A^c \in F_1$ and $A^c \in F_2$, and thus $A^c \in F_1 \cap F_2$

Let $A_1, A_2, A_3, \ldots \in F_1 \cap F_2$. Then $\forall i = 1, 2, 3, \ldots$, $A_i \in F_1$ and $A_i \in F_2$. Then

$\bigcup_{i=1}^{\infty} A_i \in F_1$ and $\bigcup_{i=1}^{\infty} A_i \in F_2$, so

$\bigcup_{i=1}^{\infty} A_i \in F_1 \cap F_2$

3. (20 points) Consider the sample space $S = [0, 1]$, the interval containing real numbers from 0 to 1. Let the probability of an interval in S be the length of the interval. Find $P(X \in [0, 1/2))$ if X is a random variable defined as $X(\omega) = \omega^2$.

$$P(X \in [0, 1/2)) =$$

$$P(\{\omega : 0 \le X < \tfrac{1}{2}\}) =$$

$$P(\{\omega : 0 \le \omega^2 < \tfrac{1}{2}\}) =$$

$$P([0, \tfrac{1}{\sqrt{2}})) = \frac{1}{\sqrt{2}}$$

4. (30 points) Let X be a geometric random variable, with probability mass function

$$P(X=k)=(1-p)p^{k-1}$$

for $k=1,2,3,\ldots$.

(a) (15 points) Find $P(X>n)$ for any integer $n\geq 0$. Your answer should be given in terms of n and the parameter p.

(b) (15 points) Compute $P(X>n+k|X>n)$ for integers $k\geq 0, n\geq 0$.

Note: You may need the formula $\sum_{i=0}^{n} x^i = \frac{1-x^{n+1}}{1-x}$.

(a) For $n>0$:

$$P(X\leq n)=\sum_{k=1}^{n} P(X=k)=\sum_{k=1}^{n}(1-p)p^{k-1}$$

$$=(1-p)\sum_{k=0}^{n-1} p^k=(1-p)\frac{1-p^n}{1-p}$$

$$=1-p^n$$

For $n=0$: $P(X>0)=1$. Thus

$$P(X>n)=1-(1-p^n)=p^n \quad \forall n\geq 0.$$

(b) $$P(X>n+k|X>n)=\frac{P(\{X>n+k\}\cap\{X>n\})}{P(X>n)}$$

$$=\frac{P(X>n+k)}{P(X>n)}=\frac{p^{n+k}}{p^n}=p^k$$