

COMER  
October 7, 2010

### **ECE 600 Exam 1**

1. Enter your name and signature in the space provided below.
2. You may not use a calculator or any other reference materials.
3. Partial credit will be given, at the discretion of the instructor.

Name:

Signature:

1. (30 points) Consider an experiment in which a die is rolled repeatedly until a six is rolled, at which point the experiment stops. The outcome of this experiment is the sequence of values rolled. Note: For this problem it is important that your notation for the sample space and the event space are consistent.
  - (a) (10 points) Define the sample space for this experiment.
  - (b) (10 points) Let  $E_n$  be the event that the total number of rolls made is  $n$ . What outcomes are in  $E_n$ ?
  - (c) (10 points) What outcomes are in  $(\cup_{n=1}^{\infty} E_n)^c$ ?

2. (20 points) Let  $F_1$  and  $F_2$  be  $\sigma$ -fields in  $S$ . Show that  $F_1 \cap F_2$  is also a  $\sigma$ -field.

3. (20 points) Consider the sample space  $S = [0, 1]$ , the interval containing real numbers from 0 to 1. Let the probability of an interval in  $S$  be the length of the interval. Find  $P(X \in [0, 1/2))$  if  $X$  is a random variable defined as  $X(\omega) = \omega^2$ .

4. (30 points) Let  $X$  be a geometric random variable, with probability mass function

$$P(X = k) = (1 - p)p^{k-1}$$

for  $k = 1, 2, 3, \dots$

(a) (15 points) Find  $P(X > n)$  for any integer  $n \geq 0$ . Your answer should be given in terms of  $n$  and the parameter  $p$ .

(b) (15 points) Compute  $P(X > n + k | X > n)$  for integers  $k \geq 0, n \geq 0$ .

Note: You may need the formula  $\sum_{i=0}^n x^i = \frac{1-x^{n+1}}{1-x}$ .