We have discussed finite and countably infinite sets. The rationals $\mathbb{Q}$ are countably infinite, but $\mathbb{Q}$ has "holes" that need to be filled. For example, the equation

$$x^2 = 2$$

has no solution in $\mathbb{Q}$.

So the real numbers $\mathbb{R}$ were created. The set $\mathbb{R}$ has no holes. Proof is beyond the scope of this class. It can be shown that $\mathbb{R}$ (and any interval in $\mathbb{R}$) is uncountable

Defn. A set is <u>uncountable</u> if it is neither finite nor countable.

An uncountable set A cannot be written as

$$A = \{x_1, x_2, \dots\}$$

Indexed Collections of sets:
the collection (or set) of sets

$A = \{A_i, i \in I\}$ for index set I is a set of elements where each element A_i is a set.

We will consider two types of I

- finite
  Can write $A = \{A_1, \ldots, A_n\}$ for finite $\cancel{\ } n \geq 1$, or
- Countable
  Can write $A = \{A_1, A_2, \ldots\}$

The union of an indexed collection A of sets is defined as

$$\bigcup_{i \in I} A_i = \{w \in S : w \in A_i \text{ for at least one } i \in I\}$$

Can write as

$$\bigcup_{i=1}^{n} A_i \quad \text{or} \quad \bigcup_{i=1}^{\infty} A_i$$

finite I countable I

Defn. The intersection of an indexed collection A of sets is

$$\bigcap_{i \in I} A_i = \{\omega \in \mathcal{S} : \omega \in A_i \text{ for every } i \in I\}$$

Can write as

$$\bigcap_{i=1}^{n} A_i \quad \text{or} \quad \bigcap_{i=1}^{\infty} A_i$$

example. $\bigcap_{i=1}^{\infty} [0, \frac{1}{i}) = \{0\}$

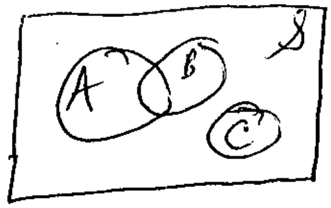
Defn. The collection $A = \{A_i, i \in I\}$ is <u>disjoint</u> if

$$A_i \cap A_j = \emptyset \quad \forall i, j, \; i \neq j$$

Defn. the collection $\{A_i, i \in I\}$ forms a <u>partition</u> of $\mathcal{S}$ if it is disjoint and

$$\bigcup_{i \in I} A_i = \mathcal{S}$$

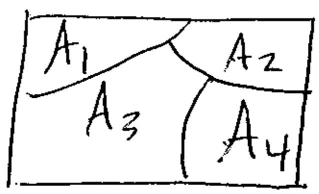
A Venn diagram is a graphical representation of a space and (some of) its subsets.



Comments:

- A Venn diagram can be very useful for gaining insight into a problem, but
  - is not a solution
  - is not a proof.
- Venn diagrams do not show probabilities

For a partition $\{A_i, i \in \{1,2,3,4\}\}$ for example,



End of Set theory review

Back to Probability Spaces

We will denote a prob. space as $(\mathcal{S}, \mathcal{F}, P)$, ~~which~~ whose elements we will now define more formally.

The Sample Space $\mathcal{S}$

$\mathcal{S}$ is a nonempty set of elements, called outcomes. Each time the random experiment is run, exactly one outcome occurs.

$\mathcal{S}$ will be either

- finite

  Ex. Toss a coin.

  Let $\mathcal{S} = \{H, T\}$

  Ex. Roll a die.

  Let $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$.

- countable

  Ex. Count cars passing

through an intersection

Let $\mathcal{S} = \{0, 1, 2, \ldots\}$

- uncountable

  Ex. Measure the operating temperature of a system (in steady state)

  $$\mathcal{S} = [T_{min}, T_{max}] \subset \mathbb{R}$$

  where T_{min}, T_{max} are fixed real numbers.

The Event Space $\mathcal{F}$

The event space $\mathcal{F}$ is the set of all subsets of $\mathcal{S}$ to which we will assign probs. Each time the experiment is run, each event either occurs or does not occur. If the outcome is in an event A, then A occurred. If not, A did not occur.