

Set theory (cont'd)

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Defn. Sets A and B are disjoint if they contain no common elements, or if

$$w \in A \Rightarrow w \notin B \text{ and}$$
$$w \in B \Rightarrow w \notin A$$

(In prob. this might also be called "mutually exclusive")

Notation:

\Rightarrow means implies or implies that

Note that \longrightarrow does not have a clear meaning

Defn. The complement of a set A

$$A^c = \{w \in S : w \notin A\}$$

Also denoted \bar{A}

Defn. The union of sets A and B is

$$A \cup B = \{w \in S : w \in A \text{ or } w \in B \text{ or both}\}$$

Defn. The ~~is~~ intersection of sets (2)
A and B is

$$A \cap B = \{w \in S : w \in A \text{ and } w \in B\}$$

Set Algebra

The operations \cap , \cup , and complement follow certain rules of algebra.

Two sets are equal if they contain exactly the same elements.

Set algebra is concerned with equality of sets.

One example, for any space S ,

$$S \cup \phi = S$$

To prove this:

Let $w \in S \cup \phi$. Then

$w \in S$ or $w \in \phi$ or both,
from defn. of \cup .

This means $w \in S$, since ϕ has no elements

Conversely, let $w \in \mathcal{S}$. (3)
Then $w \in \{a \in \mathcal{S} : a \in \mathcal{S} \text{ or } a \in \emptyset \text{ or both}\}$
since $w \in \mathcal{S}$. Thus
 $w \in \mathcal{S} \cup \emptyset$
QED

In general, can show
that $A=B$ using:

- ① let $w \in A$ and show $w \in B$, and
- ② let $w \in B$ and show $w \in A$.

Defn. A set A is a subset of
set B if
 $w \in A \Rightarrow w \in B$.

Notation: $A \subset B$

Note: Sometimes 'c' is used
for a proper subset, which
means $A \subset B$ but B contains
elements not in A ,
so $A \neq B$.

In this class, ④
 $A \subset B$ will also be
used for the case $A=B$.

Note that in this class,
any set is a subset
of itself.

Also $A=B$ iff $A \subset B$
and $B \subset A$.

A document on set theory
will be posted. It will
have many properties from
set algebra. Some important
properties:

- \cup, \cap are
 - commutative
 $A \cup B = B \cup A$
 - associative
 $(A \cup B) \cup C = A \cup (B \cup C)$
- \cap distributes over \cup
and \cup distr. over \cap
 - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

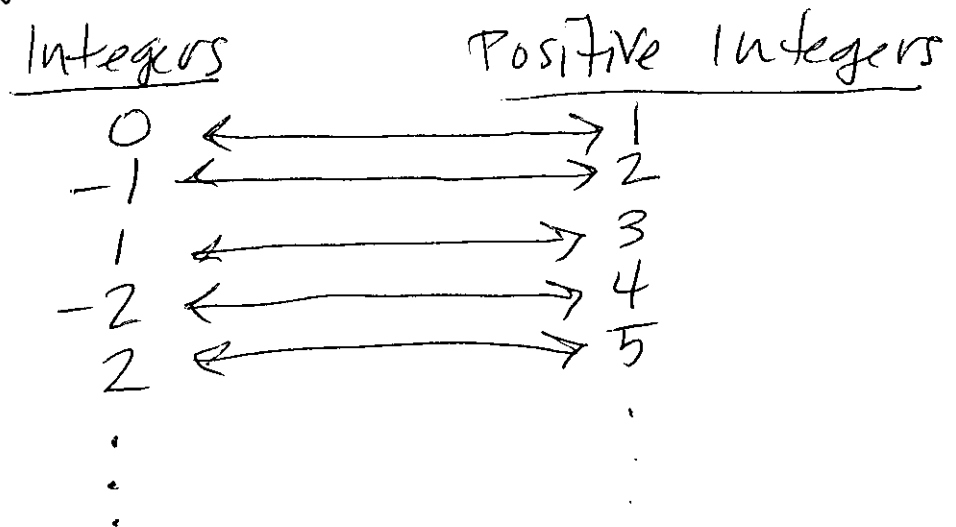
Proofs are omitted. (5)

Three types of sets:

① A set is finite if it has a finite number of elements

$$A = \{x_1, \dots, x_n\} \text{ for some finite } n.$$

② A set ~~is~~ is countably infinite, or countable, if its elements can be put into one-to-one correspondence with the integers. So the ^{set of} positive integers is countable:



Note; A finite set is not countable.

Note: In this class, (6)
countable and countably
infinite mean the same
thing. Other sources may
use countable to mean
either finite or countably
infinite.

A countable set A can be
written as

$$A = \{x_1, x_2, \dots\}$$

or

$$A = \{x_0, x_1, \dots\}$$

Also, it can be shown
that the rational
numbers \mathbb{Q} form a
countable set.

$$\mathbb{Q} = \left\{ \frac{m}{n} : m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0 \right\}$$

$n \setminus m$	0	1	2	3	...
1		1	2	3	
2		3	5		
3		4			
...					