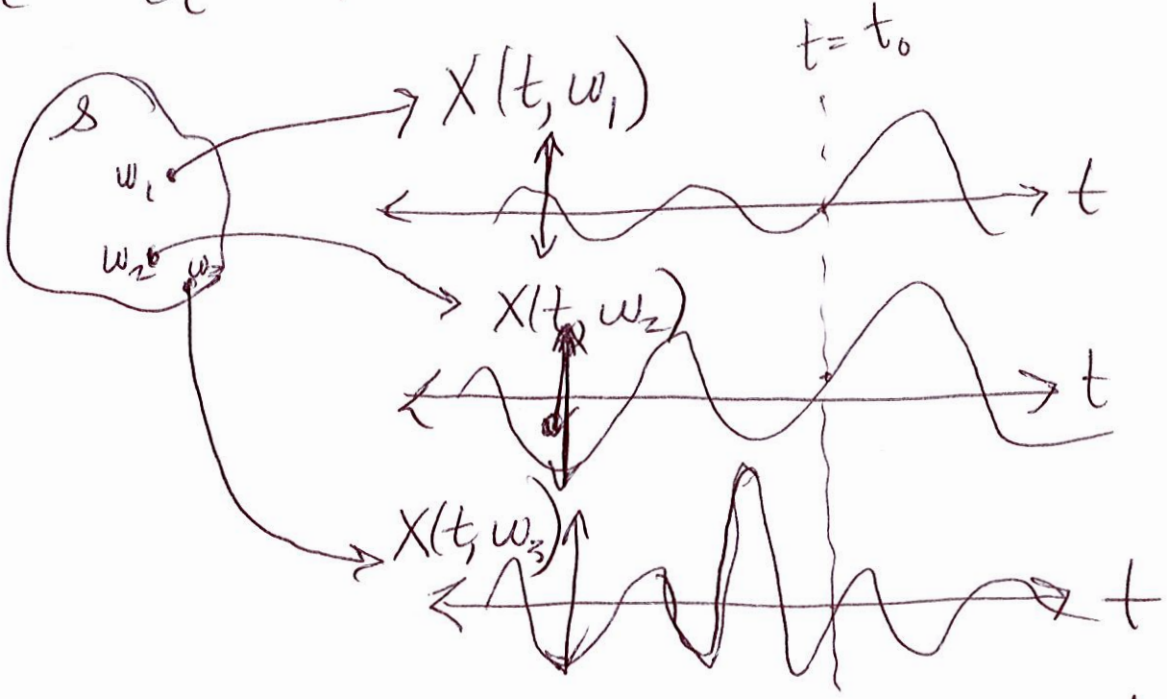
Notes from 2019 presented on chalkboard 11/17/2022

Stochastic Processes

11/19/2019

A stochastic process, or random process (rp), is a family of random variables that depends on a variable $t \in \mathbb{R}$.

Defn. A random process defined on $(S, \mathcal{F}, P)$ is a function $X(t) \equiv X(t,\omega)$ indexed by a set T



Each waveform is referred to as a sample realization, or a sample function, of the process $X(t)$

The index set T will be either uncountable or countably infinite. We will consider $T = \mathbb{R}$, $T = \mathbb{R}^+ = [0, \infty)$, and $T = \mathbb{N}$

Note that

- $X(t, \omega)$, or simply $X(t)$ is a rp over $t \in T$, $\omega \in S$
- $X(t_0, \omega)$ is a rv for any fixed $t_0 \in T$
- $X(t, \omega_0)$ is a real-valued function on T for any fixed $\omega_0 \in S$
- $X(t_0, \omega_0) \in \mathbb{R}$ for any fixed $t_0 \in T$, $\omega_0 \in S$

Four types of rvs to consider:

continuous time

① $T \subseteq \mathbb{R}$ is equal to $\mathbb{R}$ or an interval in $\mathbb{R}$; $X(t)$ is a discrete rv $\forall t \in T$

② $T \subseteq \mathbb{R}$ is equal to $\mathbb{R}$ or an interval in $\mathbb{R}$; $X(t)$ is a continuous rv $\forall t \in T$

discrete time

③ $T \subseteq \mathbb{R}$ is countable (typically $T = \{0, 1, 2, \ldots\}$ or $T = \{1, 2, 3, \ldots\}$); $X(t)$ is a discrete rv $\forall t \in T$

④ $T \subseteq \mathbb{R}$ is countable; $X(t)$ is a continuous rv $\forall t \in T$

Note that we might also have $T = \{t_1, t_2, \ldots\}$ if $X(t)$ is discrete.

Examples:

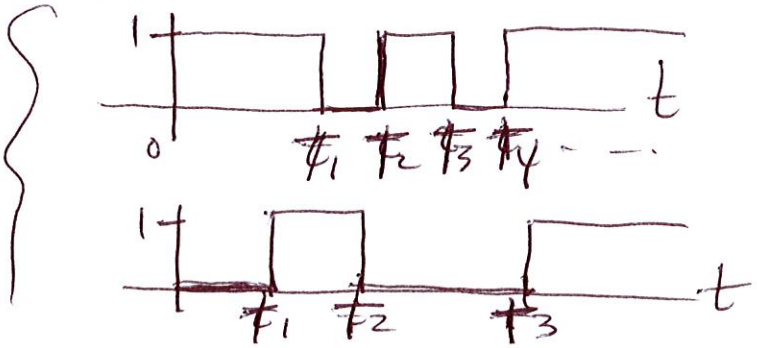
- If $X(t)$ represents the number of orders placed through Amazon.com each day, beginning with the first day of operation of the site, then $X(t)$ is a discrete-time, discrete-valued rp. We could write this as

$X(1), X(2), \ldots$, or as

$X_1, X_2, \ldots$

- A binary waveform with random transition times

Two possible sample realizations



Continuous-time, discrete-valued rp

Not that there is a corresponding discrete-time, continuous-valued rp $T_1, T_2, T_3, \ldots$ that carries the same information

- Sinusoid with random frequency

  $X(t) = \sin \Omega t$, where $\Omega = \Omega(w)$, $w \in S$, is a rv.

  Continuous-time, continuous-valued.