COMER
February 12, 2018

ECE 302 Exam 1

1. Enter your name and signature in the space provided below. YOUR SIGNATURE CERTIFIES THAT YOU WILL NOT ENGAGE IN ANY CHEATING DURING THIS EXAM.
2. You may not use a calculator or any other reference materials.

Name: SOLUTION

Signature:

No Partial Credit

You must CLEARLY select one answer for each problem.

1. (5 points) Consider events A, B, and C in an event space. Find an expression for the event that none of the events occur.
   (a) $A \cup B \cup C$
   (b) A
   (c) $A \cup B^c \cup C^c$
   (d) $A \cap B \cap C$
   (e) $A \cap B^c \cap C^c$
   (f) $A^c \cup B \cup C$
   (g) $A^c \cup B^c \cup C^c$
   (h) $A^c \cap B^c \cap C^c$

2. (5 points) Suppose that A and B are disjoint events in a probability space. If $P(A) = 0.3$ and $P(B) = 0.5$, what is the probability that both A and B occur?
   (a) 0
   (b) 0.1
   (c) 0.15
   (d) 0.2
   (e) 0.25
   (f) 0.3
   (g) 0.35
   (h) 0.4
   (i) 0.45
   (j) 0.5

3. (5 points) Which of the following statements is true in general for events A and B?
   (a) $P(A - B) = P(A) - P(B)$
   (b) $P(A) = P(B)$ implies that $A = B$
   (c) $P(A) = 0$ implies that A is the empty set
   (d) $(A \cap B)^c = A \cup B$
   (e) $A = B$ implies that $P(A) = P(B)$

4. (5 points) The sample space of a random experiment is $S = \{a, b, c, d, e\}$. You are given $P(\{a\}) = 0.2$, $P(\{a, b\}) = 0.4$, $P(\{a, b, c\}) = 0.6$ and $P(\{a, e\}) = 0.3$. What is $P(\{d\})$?
   (a) 0.0
   (b) 0.1
   (c) 0.2
   (d) 0.3
   (e) 0.4

$\{d\} = \{a,b,c,e\}^c$, so $P(\{d\}) = 1 - P(\{a,b,c,e\})$ $.6+.1=.7$

$\{a,b,c,e\} = \{a,b,c\} \cup \{e\}$, so

$P(\{a,b,c,e\}) = P(\{a,b,c\}) + P(\{e\})$

$\{a,e\} = \{a\} \cup \{e\}$, so $P(\{e\}) = .3 - .2 = .1$

5. (5 points) For the experiment in the previous problem, if $C = \{b, c\}$ and $D = \{c, d\}$, what is $P(C \cap D)$?

(a) 0.06

(b) 0.2

(c) 0.3

(d) 0.4

(e) 0.5

$P(C \cap D) = P(\{c\})$

$\{a, b, c\} = \{a, b\} \cup \{c\}$

so $P(\{c\}) = P(\{a, b, c\}) - P(\{a, b\})$

$= .2$

Note that since Problem 6 does not state whether the ~~names~~ names selected could be returned to the hat or not, I will accept either interpretation.

Limited Partial Credit

You must put your final answers in the boxes provided. Credit of either 0, 5, or 10 points will be granted for each problem. You must show your work to get credit.

6. (10 points) Three friends, Tom, Sara, and Joe, put their names in a hat, and each draws a name from the hat. The order in which the friends draw is Tom, then Sara, then Joe. Write a sample space for this experiment, if the outcome of the experiment is the three friends' names in the order in which they are drawn. Also, write the event that no one draws his or her own name.

Denote the three names T for Tom, S for Sara, J for Joe. Each outcome is a triplet of names, ordered according to the sequence of draws. Then

$$\boxed{\begin{aligned} \mathcal{S} &= \{(i,j,k) : i,j,k \in \{T,S,J\},\ i \neq j \neq k\} \\ &= \{(T,S,J),(T,J,S),(S,T,J),(S,J,T),(J,S,T),(J,T,S)\} \end{aligned}}$$

A = Event of no one drawing own name =

$$A = \{(i,j,k) \in \mathcal{S} : i \neq T, j \neq S \text{ and } k \neq J\}$$

Answer:

$$= \{(S,J,T),(J,T,S)\}$$

Note: Either form for both answers is acceptable.

7. Four marbles, numbered 1,2,3, and 4, are placed in a box. One of the marbles is drawn randomly from the box and its number, N_1, is noted (so $N_1 = 1, 2, 3$, or 4). An integer N_2 is then selected at random from the values $1, \ldots, N_1$. The outcome of this experiment is the ordered pair (N_1, N_2).

(a) (10 points) Write the sample space for this experiment.

$$\mathcal{S} = \{(i,j) : i \in \{1,2,3,4\},\ j \in \{1,..,i\}\} =$$

$$\{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),(4,4)\}$$

Answer:

Note: Either of the above forms ~~are~~ is acceptable

(b) (10 points) Write the event "Marble 2 is selected and $N_2 = 3$".

Since the ~~set~~ pair (2,3) is not in $\mathcal{S}$, the event Marble 2 ~~sel~~ selected, $N_2 = 3$ is $\emptyset$

Answer: ϕ

Partial Credit Problems

You must completely justify your solution to get full credit. Partial credit will be given at the discretion of the instructor.

8. (15 points) Prove that if an event A is a subset of an event B, then $P(A) \leq P(B)$.

We can prove this by showing that $P(B) = P(A) + \varepsilon$ for some $\varepsilon > 0$.

We can write B as

$B = (A \cap B) \cup (A^c \cap B)$, since if $\omega \in B$, then $\omega \in A \cap B$ or $\omega \in A^c \cap B$, and if ~~A~~ ω is in either $A \cap B$ or $A^c \cap B$, $\omega \in B$.

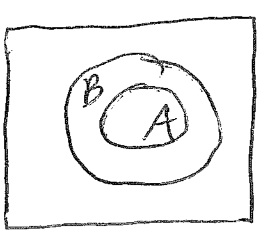
So ~~$P(A \cap B)$~~ $P(B) = P(A \cap B) + P(A^c \cap B)$

Since $A \subset B$, we know that $A \cap B = A$.

This means that

$P(B) = P(A) + P(A^c \cap B)$, so

$P(B) \geq P(A)$, since $P(A^c \cap B) \geq 0$



9. You have a pet goldfish, which you ask your roommate to feed while you are away for a few days. If your roommate does not feed the fish, the fish will die with probability 0.8. If the fish is fed, it will die with probability 0.1. (Goldfish are not known for being very robust.). Your roommate will remember to feed the fish with probability 0.9.

   (a) (15 points) What is the probability the fish will be alive when you return?

   (b) (15 points) If the fish is dead when you return, what is the probability that your roommate fed it?

(a) Let $A = \{\text{fish is alive upon return}\}$, so $A^c = \{\text{fish is dead}\}$.

Let $B = \{\text{fish was fed}\}$, so $B^c = \{\text{fish was not fed}\}$.

Then $P(B) = 0.9$, $P(A^c|B^c) = 0.8$, $P(A^c|B) = .01$

From the TPL,

$P(A) = 1 - P(A^c) =$

$1 - [P(A^c|B)P(B) + P(A^c|B^c)P(B^c)] =$

$1 - [(0.1)(0.9) + (0.8)(0.1)] = 1 - .09 - .08$

$P(A) = 0.83$

(b) $P(B|A^c) = \dfrac{P(A^c|B)P(B)}{P(A^c)}$ from Bayes' Theorem

$P(B|A^c) = \dfrac{(0.1)(0.9)}{0.17} = \dfrac{9/100}{17/100} = \dfrac{9}{17}$

$(P(A^c) = 0.17$, from Part (a)$)$