ECE 302 Homework 8
COMER

Topics: Two random variables, joint distribution functions, joint density functions, joint mass functions, independent random variables

1. The amplitudes of two signals X and Y have joint pdf

$$f_{XY}(x,y) = e^{-x/2} y e^{-y^2}$$

for $x > 0, y > 0$.

   (a) Find the joint cdf.

   (b) Find $P(X^{1/2} > Y)$.

   (c) Find the marginal pdfs.

2. The random vector (X, Y) has a joint pdf

$$f_{XY}(x,y) = 2e^{-x}e^{-2y}$$

for $x > 0, y > 0$. Find the probability of the following events:

   (a) $\{X + Y \leq 8\}$.

   (b) $\{X - Y \leq 10\}$.

   (c) $\{X^2 < Y\}$.

3. Two fair dice are rolled. Find the joint pmf of X and Y when

   (a) X is the larger value rolled and Y is the sum of the two values.

   (b) X is the smaller and Y is the larger value rolled.

4. Let $h(x,y)$ be a joint Gaussian pdf for zero-mean, unit-variance Gaussian random variables with the parameter $r = r_1$. Let $g(x,y)$ be a joint Gaussian pdf for zero-mean, unit-variance Gaussian random variables with the parameter $r = r_2 \neq r_1$. Suppose the random variables X and Y have joint pdf $f_{XY}(x,y) = [h(x,y) + g(x,y)]/2$.

   (a) Find the marginal pdfs for X and Y.

   (b) Explain why X and Y are not jointly Gaussian random variables.

5. Let X and Y be independent random variables that are uniformly distributed in $[-1, 1]$. Find the following probabilities:

   (a) $P(X^2 < 1/2, |Y| < 1/2)$.

   (b) $P(4X < 1, Y < 0)$.

   (c) $P(XY < 1/2)$.

   (d) $P(\max(X,Y) < 1/3)$.