ECE 302 Homework 4
COMER

Topics: Random variables, cumulative distribution functions, probability density functions

1. A random variable has a cumulative distribution function given by

$$F_X(x) = \begin{cases} 0, & \text{if } x < -1 \\ 0.5 + 0.5x, & \text{if } -1 \leq x < 1 \\ 1, & \text{if } x \geq 1 \end{cases}$$

   Find the probability that

   (a) $X = \frac{1}{4}$
   (b) $X > \frac{3}{4}$
   (c) $-0.5 < X \leq 0.5$

2. The cumulative distribution function for a random variable X is given by

$$F_X(x) = \begin{cases} 0, & \text{if } x < 1 \\ A[1 - \exp(-(x-1))], & \text{if } x \geq 1 \end{cases}$$

   (a) For what value of A is this a valid cdf?
   (b) What is $F_X(2)$?
   (c) Find the probability that $2 < X < \infty$.
   (d) Find the probability that $1 < X \leq 3$.

3. Find the probability density function of the random variable from the previous problem and sketch it.
   (a) Using the pdf, find the probability that $2 < X \leq 3$.
   (b) Using the pdf, find the probability that $X < 2$.

4. The probability density function of a random variable has the form

$$f_X(x) = Ke^{-2x}\mathrm{u}(x)$$

   where $\mathrm{u}(x)$ is the unit step function. Find

   (a) the value of K
   (b) the probability that $X > 1$
   (c) the probability that $X \leq 0.5$

5. Let X be an exponential random variable with parameter λ, which means that $f_X(x) = \lambda \mathrm{e}^{-\lambda x}\mathrm{u}(x)$.
   (a) For $x > 0$, find $P(X \leq x)$.
   (b) For $x_2 > x_1 > 0$, find $P(x_1 \leq X \leq x_2)$.

(c) For $x > 0$, find $P(X \geq x)$.

(d) Segment the positive real line into three equally likely disjoint intervals.

6. The median of a random variable X is defined as the value $x_m \in \mathbb{R}$ satisfying $P(X \leq x_m) = \frac{1}{2}$. Find the median of an exponential random variable X with parameter λ. Your answer should be given in terms of λ.