

**ECE 302 Homework 3**  
**COMER**

**Topics: Independent events, combinatorics, Bernoulli trials**

1. An experiment consists of picking one of two urns at random and then selecting a ball from the urn and noting its color (black or white). Let  $A$  be the event “urn 1 is selected” and  $B$  the event “a black ball is observed.” Under what conditions are  $A$  and  $B$  independent?
2. If  $A$ ,  $B$ , and  $C$  are independent events, prove that the following are also independent:
  - (a)  $A$  and  $B \cup C$ .
  - (b)  $A$  and  $B \cap C$ .
  - (c)  $A$  and  $B \cap C^c$ .
3. Out of a group of seven EEs and five MEs, a committee consisting of three EEs and two MEs is to be formed. In how many ways can this be done if
  - (a) any of the EEs and any of the MEs can be included?
  - (b) One particular EE must be on the committee?
  - (c) Two particular MEs cannot be on the committee?
4. Five blue balls, three red balls, and three white balls are placed in an urn. Three balls are drawn at random, without regard to the order in which they are drawn. After a ball is drawn, it is not replaced in the urn. Assuming that all balls in the urn are equally likely to be drawn, find the probability that
  - (a) one blue ball, one red ball, and one white ball are drawn.
  - (b) all three balls drawn are red.
  - (c) exactly two of the balls drawn are blue.
5. Consider a sequence of  $n$  Bernoulli trials with a probability  $p$  of success in each trial. For any  $0 \leq k \leq n$ , find the probability that  $k$  trials must be performed before the first success occurs.
6. Consider a batch of 100 manufactured devices, each with a probability  $p = 0.01$  of being defective, independently of the other devices. Find the probability that:
  - (a) none of the devices is defective.
  - (b) at least 95 of the devices are not defective.