ECE 302 Homework 2
COMER

Topics: probability, conditional probability, Bayes Theorem, total probability law

1. A random experiment has sample space $S = \{a, b, c, d\}$. Suppose that $P(\{c, d\}) = 3/8$, $P(\{b, c\}) = 6/8$, and $P(\{d\}) = 1/8$. Use the axioms of probability to find the probabilities of the elementary events.

2. A number x is selected at random in the interval $[-1, 2]$, where all the numbers in this interval are equally likely. This means that any event $(x_1, x_2) \subset [-1, 2]$ occurs with probability
$$P((x_1, x_2)) = \frac{|x_2 - x_1|}{|\mathcal{S}|},$$
where, for an interval I, $|I|$ is the length of the interval. Let the events $A = \{x < 0\}$, $B = \{|x-0.5| < 0.5\}$, and $C = \{x > 0.75\}$.
   (a) Find $P(A)$, $P(B)$, $P(A \cap B)$, and $P(A \cap C)$.
   (b) Find the probabilities of $A \cup B$, $A \cup C$, and $A \cup B \cup C$ using the axioms or properties derived from the axioms.

3. A die is rolled and the outcome is the value rolled. Using the counting approach to probability,
   (a) find the probability of the elementary events;
   (b) find the probability of the event A that the outcome is greater than 3, and the event B that the outcome is odd;
   (c) find the probability of $A \cup B$, $A \cap B$, and A^c.

4. A die is rolled twice and the outcome is the ordered pair containing the first value rolled and the second value rolled. Using the counting approach to probability,
   (a) Find the probability of the elementary events.
   (b) Let A be the event that the value rolled first is not less than the value rolled second, B the event that the value rolled first is 6, and C the event that the two values rolled differ by 2. Find $P(A)$, $P(B)$, $P(C)$, $P(A \cap B^c)$, and $P(A \cap C)$.

5. A number x is selected at random from the interval $\mathcal{S} = [-1, 2]$, where all numbers in this interval are equally likely. Let the events $A = \{x < 0\}$, $B = \{|x - 0.5| < 0.5\}$, and $C = \{x > 0.75\}$. Find $P(A|B)$, $P(B|C)$, $P(A|C^c)$, $P(B|C^c)$.

6. Show that $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$.

7. A candy machine has ten buttons of which one never works, two work half the time, and the rest work all the time. A coin is inserted and a button is pushed at random, with the buttons being equally likely to be pushed.
   (a) Find the probability that no candy is received.
   (b) If no candy is received, what is the probability that the button that never works is the one that was pushed?
   (c) If candy is received, what is the probability that one of the buttons that work half the time was pushed?

8. A fair coin is tossed. If it comes up heads, a single die is rolled. If it comes up tails, two dice are rolled. Given that a 3 was rolled, but you do not know if one or two dice were rolled, what is the probability that the coin came up heads?