ECE 302 Homework 10
COMER

Topics: Joint expectation, conditional distributions

1. Consider two random variables X and Y.
   (a) Write $E[(X+Y)^2]$ in terms of the moments of X, the moments of Y, and the correlation of X and Y.
   (b) Find the variance of $X+Y$ in terms of the moments of X, the moments of Y, and the correlation of X and Y.
   (c) Under what condition is the variance of the sum equal to the sum of the individual variances of X and Y?
2. Find $E[|X-Y|]$ if X and Y are independent exponential random variables with means μ_X and μ_Y, respectively.
3. Person A and Person B each toss a fair coin twice. Let X be the difference and Y the sum of the number of heads tossed by Person A and Person B. Find the correlation and covariance of X and Y, and indicate whether the random variables are independent, orthogonal, and/or uncorrelated.
4. An accident occurs at a point X that is uniformly distributed on a road of length L. At the time of the accident, an ambulance is at a location Y that is also uniformly distributed on the road. Assuming that X and Y are independent, find the expected distance between the ambulance and the point of the accident.
5. Let X be an exponential random variable.
   (a) Find the conditional cdf $F_X(x|X>t)$ and the conditional pdf $f_X(x|X>t)$, where t is a real number.
   (b) Show that $P(X>t+x|X>t)=P(X>x)$. This is called the memoryless property.
6. The input X to a communication channel is $+1$ or -1 with probability p and $1-p$, respectively. The received signal Y is the sum of X and noise N which has a Gaussian distribution with zero mean and variance $\sigma^2=0.25$.
   (a) Find the joint probability $P(X=j, Y\le y)$.
   (b) Find the marginal pmf of X and the marginal pdf of Y.
   (c) Suppose we are given that $Y>0$. Which is more likely, $X=1$ or $X=-1$?
7. The input X to a communication channel is a zero-mean, unit-variance Gaussian random variable. The channel output Y is the sum of X and a noise signal N, where N is a zero-mean Gaussian random variable with variance σ_N^2. The random variables X and N are independent.
   (a) Find the conditional pdf of Y given $X=x$. *Hint*: $Y=N+x$ is a linear function of N.
   (b) Find the joint pdf of X and Y.
   (c) Find the conditional pdf of X given $Y=y$.
   (d) Suppose that when $Y=y$ we estimate the input X by the value $x_0=g(y)$ that maximizes $P(x_0<X<x_0+dx|Y=y)$. Find x_0.