

ECE 302 Homework 1 COMER

Topics: Set theory, the sample space, the event space

1. A space S and three of its subsets are given by $S = \{1, 3, 5, 7, 9, 11\}$, $A = \{1, 3, 5\}$, $B = \{7, 9, 11\}$, and $C = \{1, 3, 9, 11\}$. Find $A \cap B \cap C$, $A^c \cap B$, $A - C$, and $(A - B) \cup B$.
2. Prove the second part of DeMorgan's Law, i.e., show that $(A \cup B)^c = A^c \cap B^c$.
3. Consider an experiment consisting of rolling a die twice. The outcome of this experiment is an ordered pair whose first element is the first value rolled and whose second element is the second value rolled.
 - (a) Find the sample space.
 - (b) Find the set A representing the event that the value on the first roll is greater than or equal to the value on the second roll.
 - (c) Find the set B corresponding to the event that the first roll is a six.
 - (d) Let C correspond to the event that the first value rolled and the second value rolled differ by two. Find $A \cap C$.

Note that A , B , and C should be subsets of the sample space specified in Part (a).

4. Let A , B , and C be events in an event space. Find expressions for the following:
 - (a) Exactly one of the three events occurs.
 - (b) Exactly two of the events occurs.
 - (c) Two or more of the events occur.
 - (d) None of the events occur.
5. Consider an experiment consisting of randomly selecting a real number r between 0 and 1. The sample space is then the interval $(0, 1)$, since the outcome can be any number in this interval.
 - (a) Find the set A representing the event that r is between 0.3 and 0.4.
 - (b) Find the set B representing the event that r is between 0.35 and 0.6.
 - (c) Find the set C representing the event that r is either less than 0.25 or greater than 0.75.
 - (d) Find $A \cap B$.
6. A system is composed of five components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcomes of the experiment be given by all vectors $(x_1, x_2, x_3, x_4, x_5)$, where x_i is 1 if component i is working and 0 if component i is not working.
 - (a) How many outcomes are in the sample space of this experiment?
 - (b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all of the outcomes in W .
 - (c) Let A be the event that components 4 and 5 have both failed. How many outcomes are in the event A .
 - (d) Write out all outcomes in the event $A \cap W$.