Some Common Random Variables

(1) Gaussian rv
(aka Normal)

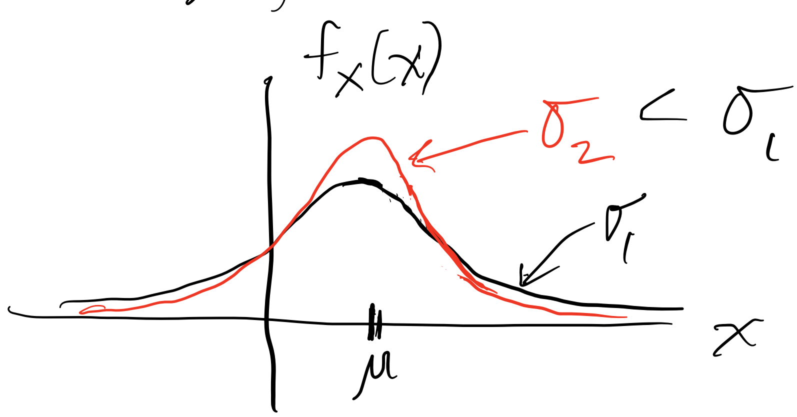
Continuous

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$\forall x \in \mathbb{R}$

$\mu, \sigma \in \mathbb{R}$, $\sigma > 0$

μ, σ are parameters



symmetric about μ

For a Gaussian

the cdf is

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] dt$$

You do not need to use the ϕ function for this class

Does not have a closed form solution.

There are tables of the function

$$\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

For general μ, σ,

$$P(X \le x) = \phi\left(\frac{x-\mu}{\sigma}\right)$$

Gaussian rv's
used to model:

- noise
- Sum of rvs
  - Will be made more ~~formal~~ with the Central Limit Theorem
- random variables with histograms looking like a "bell curve"

Notation:

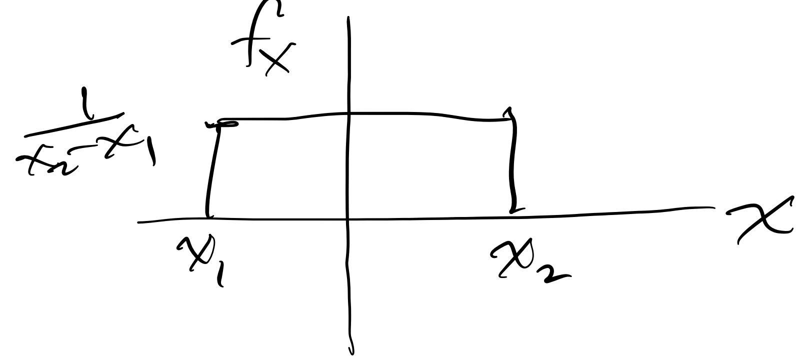
$$X \sim N(\mu, \sigma^2)$$

② Uniform

– Continuous form

$$f_X(x) = \begin{cases} \dfrac{1}{x_2 - x_1} & x_1 \le x \le x_2 \\ 0 & \text{otherwise} \end{cases}$$

for some $x_1 < x_2$, $x_1, x_2 \in \mathbb{R}$



– Discrete form

$R_X = \{x_1, \ldots, x_n\}$

The pmf is

$$p_X(x) = \frac{1}{n} \quad \forall x \in R_X$$

Used to model:

- Equally likely values of X

For the continuous case, this means $P(X \in (a,b))$ depends only on $|b-a|$ for $x_1 \le a < b \le x_2$:

$$P(X \in (a,b)) = \frac{|b-a|}{|x_2 - x_1|}$$

③ Bernoulli rv

Discrete

$$R_X = \{0, 1\}$$

$p_X(0) = p$

$p_X(1) = 1-p$

for parameter

$0 \le p \le 1.$

(5) Binomial rv

Discrete

$R_X = \{0, \dots, n\}$

$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k},$

$k = 0, \dots, n$

parameters $p, n,$

n finite

Can be used to

model the number of successes in n Bernoulli trials