We have learned about the pdf and cdf, and have the result:

$$P\left(X \in A\right) = \int_A f_X(x)\, dx \qquad A \subset \mathbb{R}$$

"is a subset of"

Note: "is an element of"

Some common forms of A:

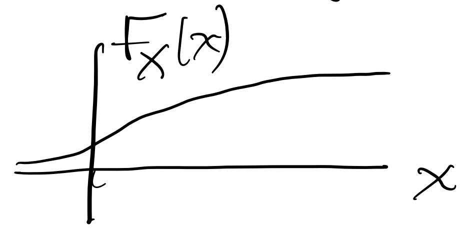
- $A = (-\infty, x]$
- $A = (-\infty, x)$
- $A = (x, \infty)$
- $A = [x, \infty)$
- $A = (x_1, x_2)$

where $x, x_1, x_2 \in \mathbb{R}$;
$x_1 < x_2$

Note that every rv we will encounter in this class will have a cdf and pdf. For one type of rv, there is another function that is more often used in practice.

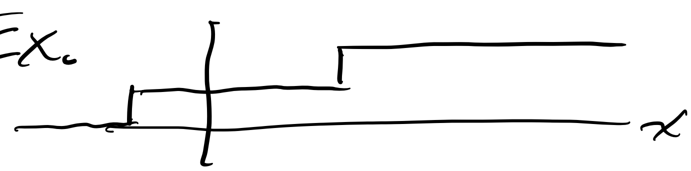
Types of Random Variables

Defn. A rv X having a cdf that is continuous everywhere ($\forall x \in \mathbb{R}$) is a <u>continuous rV</u>.



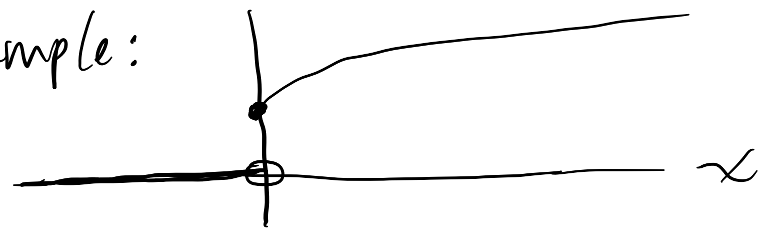
Defn. A rv X whose cdf is piecewise constant, with a finite or countably infinite number of jump discontinuities is a <u>discrete rV</u>.

Ex.



Note that if a rv is continuous everywhere, except for a finite or countably infinite set of jump discontinuities, and is not discrete, then it is a <u>mixed rv</u>

Example:



Mixed rvs sometimes appear in practice, but we will not see them in this class.

Note that for a continuous rv, at each $x \in \mathbb{R}$,

$$P(X = x) = F_X(x) - F_X(x^-) = 0,$$

since the cdf is continuous everywhere (and thus $F_X(x^-) = F_X(x)$)

One consequence of this can be seen as follows:

$$P(X \leq x) = P(\{X < x\} \cup \{X = x\}),$$

and since $\{X < x\}$ and $\{X = x\}$ are disjoint,

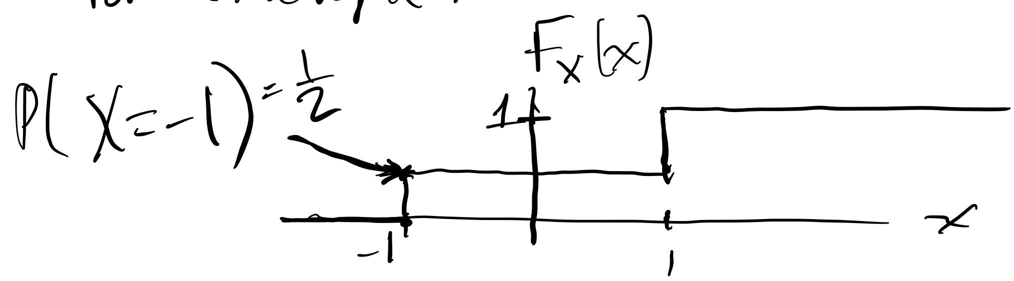
$$P(X \le x) = P(X < x) + P(X = x) = P(X < x)$$

Since $P(X = x) = 0$ $\forall x \in \mathbb{R}$ if X is a continuous rv

So for a continuous rv, can interchange $\le$ and $<$, and $\ge$ and $>$

For a discrete rv, since the pdf is the derivative of the cdf, this leads to a pdf with δ-functions.

For example:



The pdf can be written as

$$f_X(x) = \frac{1}{2}\delta(x+1) + \frac{1}{2}\delta(x-1), \quad \forall x \in \mathbb{R}$$

Dirac delta fn.

Instead of using the pdf for a discrete rv, usually use:

The Probability Mass Function

Defn. For a discrete rv X, the probability mass function, or pmf, or simply mass function, is

$$p_X(x) \equiv P(X = x), \forall x \in \mathcal{R}_X \subset \mathbb{R}.$$

where $\mathcal{R}_X$ is the set of points where F_X has a discontinuity.

For the example above,

$\mathcal{R}_X = \{-1, 1\}$

Unless otherwise specified, use the pmf instead of the cdf, pdf, for a discrete rv.

Some properties of the pmf:

If a function satisfies these two properties, it is a valid pmf for some rv X

1. $p_X(x) \geq 0, \forall x \in \mathcal{R}_X$
2. $\sum_{x \in \mathcal{R}_x} p_X(x) = 1$
3. If $A \subset \mathcal{R}_X$, then

$$P(X \in A) = \sum_{x \in A} p_X(x),$$

since $\{X \in A\} = \cup_{x \in A}\{X = x\}$

Some practical advice for working with rvs:

1. Is the rv continuous or discrete?

- If continuous, use cdf or pdf
- If discrete, use pmf
- If it cannot be determined from the problem statement, use cdf or pdf.

② What is the problem asking for?

- The prob. of some event: you might use

$$P(X \le x) = F_X(x),$$

$$\int_A f_X(x)\,dx, \text{ or}$$

$$P(X = x) = p_X(x).$$

You may need to use set algebra and axioms/properties of P to find the prob.

— The cdf, pdf, or pmf of a rv, your answer must be a function

The function must be defined on

- all $x \in \mathbb{R}$ for a cdf or pdf
- all $x \in R_X$ for a pmf.