Have introduced the mean, autocorrelation, and autocovariance functions of a rp $X(t)$

Note that $\exists$ can be shown that

$$C_{XX}(t_1, t_2) = R_{XX}(t_1, t_2) - \mu_X(t_1) \cdot \mu_X(t_2)$$

We might also characterize a rp using nth order cdfs, pdfs, or pmfs for $n = 1, 2, \ldots$

For example, for a fixed t, the first-order cdf of $X(t)$ is $F_{X(t)}(x) = P(X(t) \leq x)$,

$\forall x \in \mathbb{R}$

For a different t, this cdf would in general be a different function.

For fixed t_1, t_2, the second-order cdf is

$$F_{X(t_1)X(t_2)}(x_1, x_2) = P\left(X(t_1) \le x_1, X(t_2) \le x_2\right)$$

for all $x_1, x_2 \in \mathbb{R}$.

The pdf in this case would be

$$f_{X(t_1)X(t_2)}(x_1, x_2) = \frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2}\, F_{X(t_1)X(t_2)}(x_1, x_2)$$

If $X(t)$ is a discrete-valued rp, use

$$P(X(t_1) = x_1, X(t_2) = x_2),$$
$$\forall x_1 \in \mathcal{R}_{X(t_1)}, x_2 \in \mathcal{R}_{X(t_2)}$$

joint pmf $p_{X(t_1)X(t_2)}$ of $X(t_1), X(t_2)$

Defn. A rp $X(t)$ is a Gaussian

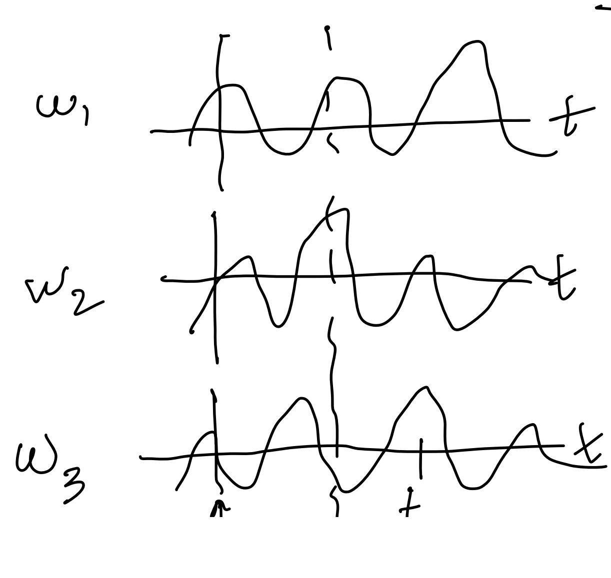
rp if $\forall n \in \mathbb{N}$ and any $t_1, .., t_n$, the rvs $X(t_1), X(t_2), \ldots, X(t_n)$ are jointly Gaussian.

(You do not need to know the nth order Gaussian pdf, but it has a similar form to the 2nd order)

Stationary of Random Processes

Fitting models to rps in practice can be very difficult.

One concept that can help with this is stationarity.



Would $E[X(t_1)]$ change if I moved the time origin?

$t=0 \quad t=0$

Also, does $f_{X(t_1)}$ change if the time origin changes?

These are questions addressed by stationarity.

Two types of stationarity

① Strict-sense stationarity (SSS)

A rp $X(t)$ is SSS if

$$F_{X(t_1)\cdots X(t_n)}(x_1, .., x_n) =$$
$$F_{X(t_1+\alpha)\cdots X(t_n+\alpha)}(x_1, \ldots, x_n)$$

$\forall \alpha$, $\forall n \in \mathbb{N}$, $\forall t_1, .., t_n$; $\forall x_1, .., x_n$