The Central Limit Theorem

The Central Limit Theorem (CLT) basically says that the sum $X_1 + X_2 + \cdots + X_n$ of iid random variables will converge to a Gaussian rv as $n \to \infty$.

More specifically, if $X_1, \ldots, X_n$ is a sequence of iid rvs, each with finite mean and finite variance, then

$$Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

converges to a Gaussian rv.

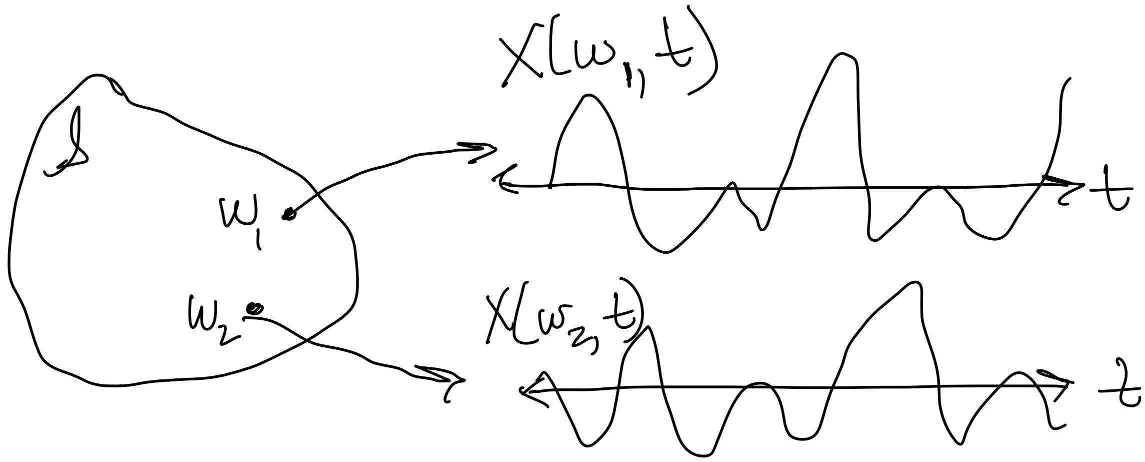
Note that we have seen that if X_1, X_2 are iid exponential rvs, then the pdfs look something like this:

So even with just $n=2$, we can already see more of a "Gaussian-like" pdf in the exponential case.

Random Processes

A random process (rp) is a family of rvs indexed by a variable t. We will refer to t as "time".

Conceptually, when we run a random experiment, we get an outcome $\omega \in S$. This ω is mapped to a waveform $X(\omega, t)$ that depends on <u>both</u> ω and t



and so on for other w.

The notation $X(w, t)$ is usually shortened to $X(t)$, but it should be understood that $X(t)$ also depends on w.

Two types of rp:

- discrete-time, for which $t \in \mathbb{N} = \{1, 2, \ldots\}$. In this case we usually write $X(t)$ as $X_1, X_2, \ldots$

- continuous-time, for which $t \in \mathbb{R}$ or

  $t \in [0, \infty)$

Note that for a rp $X(t)$:

- $X(w, t_0)$ is a rv for a fixed t_0
- $X(w_0, t)$ is a realvalued function of the variable t for a fixed $w_0 \in S$. This is called a "sample realization" of the rp $X(t)$
- $X(w_0, t_0)$ is a real number for fixed w_0, t_0

For each of the two types of rp, there are two subtypes:

- $X(t)$ is a continuous, or continuous-valued, rp if $X(t)$ is a continuous rv for all t
- $X(t)$ is a discrete, or discrete-valued, rp if $X(t)$ is a discrete rv for each t.

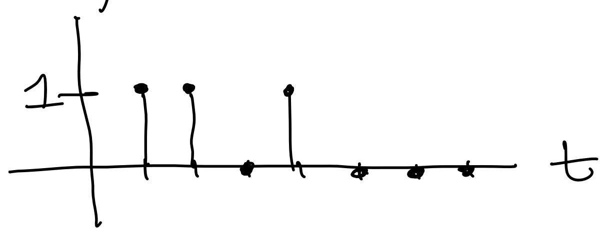
We will not consider any other cases in this class.

Some examples of rps:

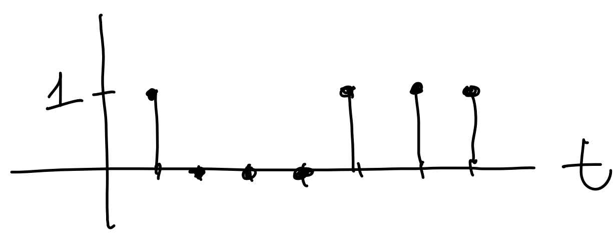
1. Let $t \in \mathbb{N}$ and let $X(t)$ be a Bernoulli rv with $p = \frac{1}{2}$ for each t.

Some sample realizations

$X(\omega_1, t)$



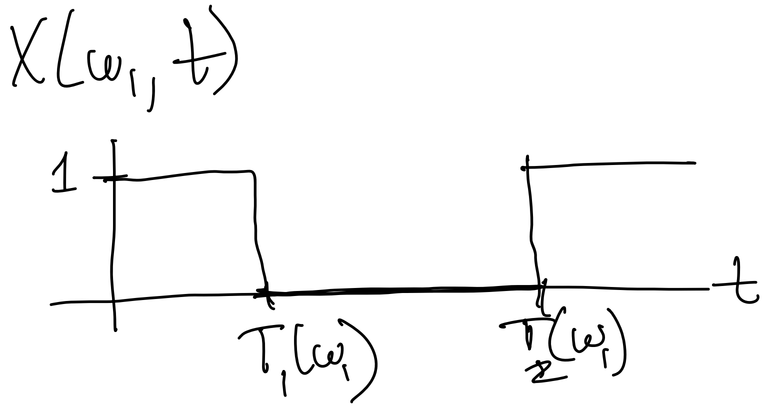
$X(\omega_2, t)$

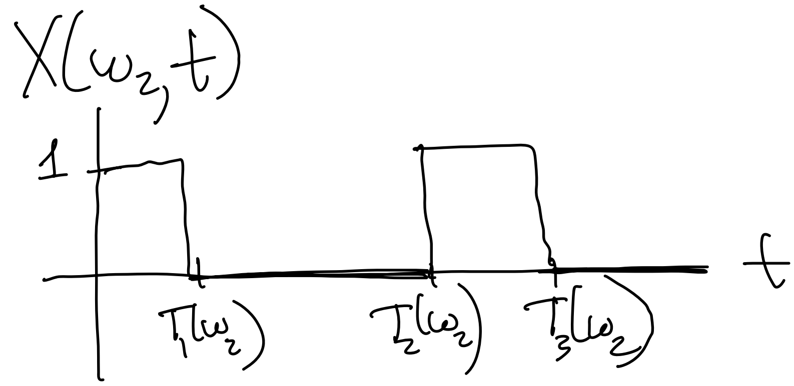


This process is discrete-time, discrete valued.

② Binary waveform with random transition times

Sample realizations





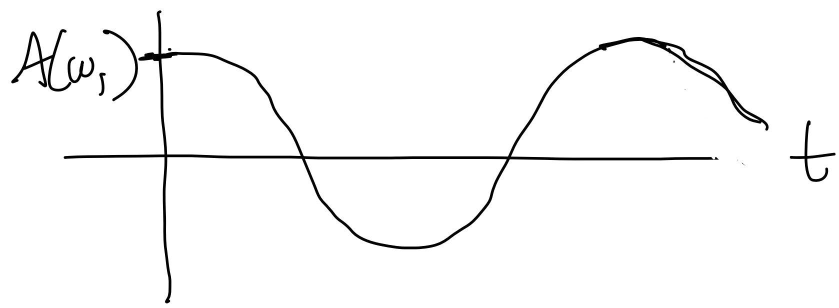
Continuous-time, discrete-valued process

③ Sinusoid with random frequency and amplitude

$$X(t) = A \cos \Omega t$$

where A, Ω are rvs

$X(\omega_1, t)$



$X(\omega_2, t)$