Gibbs-Sampling-based Optimization for the Deployment of Small Cells in 3G Heterogeneous Networks

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Abstract—The growing popularity of mobile data services has placed great demands for wireless cellular networks to support higher-throughput. One way to meet the rapidly growing traffic demand is through heterogeneous network deployment, which uses a mixture of macro cells and small cells (i.e., micro-/ pico-cells) to further enhance the spatial reuse and thus improves network throughput. In this paper, we propose a Gibbs-Sampling based optimization method for finding the optimal deployment of a given number of small cells in 3G networks. The Gibbs Sampling method intelligently balances two potentially conflicting considerations of placing small-cell BSs close to hotspots and avoiding interference with the macro-cell BSs & other small cell BSs. We show that it converges to the deployment decision with the maximum total system throughput with high probability. We also describe two low-complexity algorithms, the greedy EcNo and the greedy hotspot algorithms. Both algorithms are widely used in industry and can be used as the benchmark for comparing our Gibbs sampling-based (GSB) design. We have conducted extensive simulations based on real traffic traces from the 3G data network. Our numerical results show that the GSB placement outperforms the greedy solutions. The GSB approach produces 10% higher throughput and 30% higher off-loading factor than the greedy solutions. Since the cost of deploying small nodes could be expensive and each city may need a large number of small nodes, the proposed results thus represent significant cost savings when compared to the existing greedy solutions.

I. INTRODUCTION

The proliferation of smart phones and tablets has resulted in much higher data traffic demands on the cellular networks. It is now common to use 3G/4G cellular networks and/or Wi-Fi to provide multimedia services to mobile users. Many of the mobile applications also have stringent Quality-of-Service (QoS) requirements. Between 2007 and 2009 an unprecedented 5000% increase in data traffic has been witnessed mainly due to the iPhone [1]. In the USA, nearly 100 million people have subscribed to wireless data plans and used smartphones as one of their main portals for accessing Internet [2]. It is projected that by 2014 an average cell phone user will consume 1GB data per month, a factor of 100 higher than today’s average per-user data consumption [3], and the average wireless data connection speed will surpass 1 Mbps in 2014 [4].

To meet the ever-growing demands of higher throughput and better quality-of-service, cellular carriers have employed different ways to expand the network capacity, including the use of smart antenna, better scheduling and network coding algorithms [5], [6], high-order modulation, and sectorization of the cells. One promising approach is to use small low-power cells1 (sometimes termed micro/ pico base-stations) [7] that complement the regular base stations (sometimes termed the macro base-stations) to further enhance the spatial reuse and improve throughput.

It is believed that the next generation of cellular networks will consist of a mixture of macro cells2 and small micro/ pico cells [3], which is commonly referred to as Heterogeneous Networks (HetNet) (see Fig. 1). The small cells can operate either in the same channel as the macro nodes (the co-channel mode), or in different channels than the macro cells (the dedicated small cell carrier mode). By leveraging upon the new frequency reusage opportunity in an HetNet environment, mobile carriers can maximize the spectrum efficiency and provide higher network capacity to meet the demands for future wireless communications.

How to deploy the small nodes in an HetNet environment has become an interesting problem. The main goal is to enhance the throughput performance for each user

1We will use “small node” to denote the small-cell base station interchangeably.
2The traditional base station in the 3G network is usually named as Node-B. To distinguish from the small nodes, we will use the term “macro node” to refer to the Node-B.
on the boundary of the macro cells, for which the small nodes will be placed. An equally important metric is how much traffic of the original macro nodes can be offloaded to the newly placed small nodes. One can easily see that the small node placement problem depends on the network configuration and the traffic demand profiles for different geographical locations. For example, a reasonable scheme should place the an appropriate amount of small nodes in the area with high traffic demands, but at the same time not being too close to the macro nodes so that we can better offload the traffic from the macro nodes. The development of wireless localization techniques and the enhanced location awareness at user equipment (such as smartphones) enables the wireless carrier to construct the geographic distribution of data traffic. This information is particularly useful for the optimization of the deployment of small cells. Small nodes can be placed close to the hotspots so that data offload capability is maximized.

[7] has investigated on how to use pico cells to improve the performance in UMTS networks under the simple setting of 1 macro and 1 pico node. In contrast, our work considers the scenario where multiple small nodes will jointly offload the traffic from multiple macro nodes. We will also study the question how many small nodes one should place under the given geographical user traffic profile. [8] claims that in LTE network, pico nodes deployed at the macro cell edge will improve performance more than pico cells in the cell center. They also study the importance of placing pico nodes in the hotspot. In our work, we propose the Greedy hotspot algorithm as a benchmark to compare with the proposed GSB algorithms. To our best knowledge, this work is the first solution for the placement of small-cell base-stations that jointly optimizes the locations of multiple small nodes with the goal of maximizing any given network utility function. The closest solution in the exiting literature is the femtocell deployment problem studied in [9]–[11]. However, those results do not apply to our setting for the following reason, [9], [10] discuss the problem of femtocell placement in a single building. The goal is to minimize the power assumption for the mobile handsets while covering “all the service areas in a building.” [11] aims to minimize the coverage holes and pilot transmission power, which focuses on how to adjust the power control, in order to optimize the public coverage space. However, in our work the area of interest is already covered by macro nodes so we do not have the “coverage constraint”. Instead, we focus on how to best offload data traffic from macro nodes to small nodes, so that the congestion at the macro cells is relieved. The offloading effect is not studied in [9]–[11]. Moreover, our deployment highly relies on the traffic demand profiles for different geographical locations, which has not been considered in any previous work including [9]–[11]. When considering the traffic geographical distribution, increasing the coverage area only does not necessarily improve the throughput in the area of interest. [9], [10] formulate the femtocell placement problem as an integer convex problem. We allow for arbitrary network utility functions, including throughput and the traffic offloading factor, while taking into account the SINR model with co-channel interference from all the small nodes. Another difference from [9], [10] is that they try to minimize the maximum uplink transmission power, while our work focuses on adjusting the small node locations to improve the downlink transmission speed. [11] assumes networks with fixed transmitter locations and determines the transmitter’s pilot channel power. However, in some scenarios, especially when data traffic demand is high, even if the network configuration is optimized, it is still difficult to meet the capacity requirements [3]. In contrast, we are focusing on the proactively optimizing the small nodes locations, to achieve higher network throughput. [11] only implicitly considers decreasing the interference by reducing the coverage overlap. In our work, we explicitly include interference in calculating the SINR when determining the throughput.

The main contributions of this work are

- We incorporate the traffic demand profiles for different geographical locations into the optimization for placement of small nodes, which has not been considered in any previous work. We propose Gibbs sampling algorithms to optimize the total throughput of all users in the area of interest.
- Empirically, we have conducted extensive simulations based on real traffic traces directly from existing 3G networks. Our results show that Gibbs sampling converges quickly to the optimal solution and the resulting throughput is 10%-15% better than that of the greedy algorithm.

II. THE SETTING

We focus on the downlink transmission in the area centered by several adjacent macro cells. For ease of

\[\text{Too many small nodes placed nearby will create strong co-channel interference and thus decrease the throughput.}\]

\[\text{Hotspot indicates an area with high user density.}\]
Among the $N$ cell (the center mini-cell), we use the data traffic density. For example, for the 5-th mini-cell, we use $c_5$ MB per hour. Since any mobile is associated to either a macro node or a small node, we use $U_n$ to denote the number of users associated with node $n$ for all $n = 1, \cdots, N_m + N_p$. For any user $i$, we use $n(i)$ to denote the index of the BS with which user $i$ is associated. We sometimes call $n(i)$ the serving BS of user $i$. We use $U = \sum_{n=1}^{N_m+N_p} U_n$ to denote the total number of users in the area of interest.

Let $l_{n,i}$ denote the path gain from cell $n$ to user $i$, for all $n = 1, \cdots, N_m + N_p$ and $i = 1, \cdots, U$. In the HSDPA standard, each node is given some power constraint. We assume the power constraint for each small node is $P_s$ Watt. For the macro nodes, the power constraint may vary and we assume that the power constraint for macro node $n$ is $P_m(n)$ Watt. Given the maximum power constraint, each node only uses a certain fraction of the constraint when this node is not serving any user since it still needs to continuously broadcast some control information. We denote that fraction by $1 - h_f$ where $h_f$ stands for total HSDPA power fraction, i.e., the fraction of power that can be assigned to High Speed Downlink Shared channel (HS-DSCH). A typical $h_f$ value is 50% for a marco node and 80% for a small node. Let $P_n$ (Watt) denote the transmission power of node $n$, for $1 \leq n \leq N_m + N_p$. Therefore, we have for any small node $n = N_m + 1, \cdots, N_m + N_p$,

\[
P_n = \begin{cases} 
P_s & \text{if at least one user is associated} \\
P_s \times (1 - h_f) & \text{if } U_n = 0 
\end{cases}
\]

and for any macro node $n = 1, \cdots, N_m$

\[
P_n = \begin{cases} 
P_m(n) & \text{if } U_n \geq 1 \\
P_m(n) \times (1 - h_f) & \text{if } U_n = 0 
\end{cases}
\]

To calculate the power received by the users, we need to study the path-gain for the propagation. The average propagation gain between small node $n$ and the mobile user is calculated according to the models in [12], [13]. The transmission power $P_n$ and the path gain $l_{n,i}$ will be used subsequently to compute the signal to interference ratio and the throughput of individual users.

We now discuss how to compute $n(i)$, the serving node of user $i$. In HSDPA, each user $i$ measures Common Pilot Channel (CPICH). Then user $i$ chooses the node that has the largest Received Signal Code Power (RSCP) value (with unit Watt), which is the power level received from the pilot channel of a node. The RSCP value of user

Fig. 1. HetNet is comprised of macro nodes and small nodes. Data traffic on the edge of macro cells are offloaded to the small cells, so that the performance on the macro-cell boundary area is improved and traffic load for the macro cell is alleviated.

Fig. 2. Illustration of the mini-cells, the corresponding data traffic density, and three coexisting macro nodes.

exposition, we assume that the area of interest is rectangular with length $L$ and width $W$ (meters). We divide the area into $N_s \times N_t$ number of rectangular mini-cells. For any mini-cell, the network designer may choose to place a small node (transceiver) in the center of the given mini-cell. Consider the example in Fig 2, for which the area of interest is evenly divided into $3 \times 3$ mini-cells. We use #1 to #9 to label the 9 mini-cells in Fig. 2, respectively. For the $k$-th mini-cell, we use $c_k$ (MB/ Hour) to represent the data traffic density. For example, for the 5-th mini-cell (the center mini-cell), we use $#5 : 0.5$ to indicate the corresponding data traffic density is 0.5 MB per hour.

Suppose that there are $N_m$ macro nodes nearby and our goal is to place $N_p$ small nodes in the mini-cells. Among the $N_m + N_p$ transmitters of interest, we use the indices $n = 1, 2, \cdots, N_m$ to denote the existing macro base stations and use the indices $n = N_m + 1, N_m + 2, \ldots, N_m + N_p$ to denote to-be-placed small nodes. Since any mobile is associated to either a macro node or a
i from a node n is defined as

\[ \text{RSCP}_i^n = \text{Pilot}(n) \times l_{n, i}, \]

where \( \text{Pilot}(n) \) (in Watts) denotes the power of the pilot channel for node n. We assume that for each small node, it uses 10% of its power as the pilot power. The pilot power for macro node is from the existing 3G network.

In summary, given the traffic profile \( c_k \) of each individual mini-cell k, our goal is to find the optimal placement of the \( N_p \) small nodes that maximizes the the throughput of users in the area of interest, based on the long term data traffic geographic information. For the purpose of non-disclosure, we could not explicitly present the data details. Some of the simulation results may only reflect the relative ratios but not the real values.

### A. Throughput Calculation

For the UMTS HSDPA system, \( W = 38400000 \) is the WCDMA chip rate. Let \( N_0 = 3.9811e^{-21} \) watts/Hz be the thermal noise power spectral density. Suppose \( P_{n, i}^a \) is the total power that user i receives, and \( P_{n, i}^b \) is the power that user i receives from the serving node n. Recall that \( P_n \) is the total power used by node n, and \( l_{n, i} \) is the path-gain between node n and user i, then we have

\[ P_{n, i}^a = WN_0 + \sum_{k \neq n(i)} P_k l_{k, i}, \quad (3) \]

and

\[ P_{n, i}^b = P_n l_{n, i}. \quad (4) \]

For user i, we use \( \sigma^i \) to denote the ratio of interference and noise power to the service power, which is given by

\[ \sigma^i = (P_{n, i}^a - P_{n, i}^b) / P_{n, i}^b. \quad (5) \]

We use \( \text{SINR}_i \) denotes the SINR for user i and \( \text{SINR}_{db} \) denotes the SINR for user i measured in db. Apparently \( \text{SINR}_{db} = 10 \log(\text{SINR}_i) \). Recall that \( hf \) denotes the fraction of the total cell power available for HS-DSCH. Like the four-parametric Weibull functions in [14], we use the following approximation

\[ \text{SINR}_{db} = 10 \log_{10}(hf) + a - be^{-(\sigma^i)^d}, \quad (6) \]

where we set \( a = 9.23, b = 55.78, c = 1.62, \) and \( d = -0.22. \)

Users can estimate the channel quality and map the SINR to the Channel Quality Indicator (CQI). We follow the formula for mapping SINR to the CQI for user i in [15] as

\[ \text{CQI}_i = \min(30, \max(0, \left[ \text{SINR}_{db}/1.02 + 16.62 \right])). \quad (7) \]

We can easily see that each CQI is an integer ranging between \([0, 30]\). We can derive the peak throughput \( T_{\text{peak}} \) for user i by looking up in Table I. We assume that for each node n, all users that associate with node n equally share its bandwidth.

### III. Optimization of Small-Nodes Deployment

Finding the optimal locations for small-cell deployment is non-trivial problem. The complexity for computing the optimal locations grows exponentially as the number of mini-cells increases. In this section, we first introduce greedy approaches to deploy the small nodes in the area of interest.

#### A. Greedy EcNo Algorithm

In the first greedy algorithm, the discussions are based on the two concepts RSCP and EcNo. Recall that RSCP is the power level received from the pilot channel of a cell. The mobile can compare the RSCP values from different cells, and make cell association decisions. EcNo means the received energy per chip (Ec) of the pilot channel divided by the total noise power density (No). The better the EcNo value is, the better the signal from a cell can be distinguished from the overall noise. In this algorithm, we greedily select the locations for small nodes based on the EcNo values.

#### § Greedy EcNo

1. Pick the first small node, place it in mini-cell 1, and calculate the EcNo value for all users

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<td>30</td>
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2: Move this small node to mini-cell 2 and recalculate the EcNo value.
3: Repeat this process (from Line 1 to Line 2) for all mini-cells. Find the mini-cell which gives the largest total EcNo values for all users, place the small node in that mini-cell. Associate all users to either the small node and the macro nodes according to their RSCP values.
4: Repeat the process from Line 1 to Line 3 for all the other small nodes.

B. Greedy Hotspot Algorithm

Small nodes are supposed to offload traffic from the macro nodes. As a result, the throughput performance in the macro cells will improve since fewer users will share the bandwidth in the macro network. Further, users covered by the small nodes will also enjoy better transmission conditions and hence higher data rates. To best offload traffic from the macro cells, it is intuitive to deploy small nodes in the vicinity of the traffic hotspots, as in the following algorithm.

§ Greedy Hotspot
1: Pick small node \( n \)
2: From all the mini-cells which does not have any small node deployed, choose the one with the highest data traffic volume, place node \( n \) in the mini-cell.
3: Place all the small nodes sequentially

C. Discussion

Greedy EcNo and Greedy hotspot algorithms are fast to implement. However, their performances could be poor. Greedy EcNo algorithm greedily deploys the small nodes based on signal quality only, while Greedy hotspot algorithm acts upon the traffic profile only. Suppose that we are trying to deploy two small nodes in the area shown in the example of Fig 2. Apparently, Greedy hotspot algorithm would choose mini-cell 9 and mini-cell 6 since they have the largest data traffic density. Then users close to mini-cells 1, 4, and 7 may not be offloaded to the small nodes. For greedy algorithm, it picks mini-cell 5 first, since placing a small node right in the center can result into the maximum total EcNo values. Later mini-cell 9 will be chosen for the second small node. Thus, many users may be far from those mini-cells and their traffic could not be offloaded to the small nodes. A better deployment would be mini-cells 4 and 6. Not only the hotspots are close to a small node, also the two small nodes are relatively far away so that the interference could be reduced. This example illustrates the importance to take the advantage of both signal quality information and traffic density profile.

IV. Gibbs Sampling for Optimization of Small Node Location

We next propose an optimization algorithm based on Gibbs Sampling. Our goal is to optimize the small nodes locations so that the total utility for the users in the area of interest can be improved. More specifically, we are trying to increase the total throughput for all users within the area of interest. The basic idea is to use randomization to solve it. First we place all small nodes randomly: each small node randomly chooses one mini-cell and associates with nearby users. Then, in each iteration, each of the small nodes one-by-one switches to a neighboring mini-cell according to a probability distribution, and re-assocates with users. This location update process forms a stochastic process. The steady state distribution is carefully chosen so that the optimal solutions are the most likely to occur.

More specifically, we define the position vector
\[ \mathbf{s} = (s_{N_m+1}, s_{N_m+2}, ..., s_{N_m+N_p}). \]
to represent the locations of all small nodes. We use \((\mathbf{s}/n)\) to represent the small nodes locations except small node \( n \), and \((\mathbf{s}/n, b_n)\) to denote the vector
\[ (s_{N_m+1}, ..., s_{n-1}, b_n, s_{n+1}, ..., s_{N_m+N_p}). \]

Let \( S \) denote the feasible set, that is, the set of all possible small node placements. Let \( S^* \subseteq S \) be the set of optimal solutions.

We define the global utility \( \mathcal{U}: S \rightarrow \mathbb{R} \), as the summation of user throughputs in the area of interest. Recall that, for each user \( i \), we can estimate the SINR and map the SINR to the CQI value to derive the peak throughput \( T^i_{\text{peak}} \) for user \( i \). Further recall that we use \( U_n \) to denote the number of users who are associated with node \( n \), and \( n(i) \) to denote the node that user \( i \) has been associated with. The average throughput \( T^i_{\text{avg}} \) for user \( i \) can be obtained by \( T^i_{\text{peak}}/U_n(i) \).

Without loss of generality, we assume that the time to begin is \( t = 0 \) and the initial placement of small nodes can be given or randomly generated. We expect to generate a continuous-time Markov chain \( X(t) \in \mathbb{R}_+ \) that has a stationary distribution \( \pi: S \rightarrow [0, 1] \) given by
\[ \pi(\mathbf{s}) = \frac{\exp(\gamma \mathcal{U}(\mathbf{s}))}{\sum_{\mathbf{s}' \in S} \exp(\gamma \mathcal{U}(\mathbf{s}'))}. \]
where $\gamma > 0$ is a fixed parameter. This distribution is called the Gibbs measure [16]. The expression of Gibbs measure is very intuitive: it favors location vector $s$ with larger values of utility $\mathcal{U}(s)$. The Gibbs sampler to be presented below drives the small node placement to a steady state distributed according to (8).

The transitions in $X(t)_{t\in \mathbb{R}^+}$ correspond to a change in small node locations. Note that the locations of macro nodes are fixed. For ease of illustration, we assume only one small node changes its location at a time. After all small nodes take turns to update their locations, we call this as an iteration.

We use the method of Gibbs sampling [16] to generate the Markov chain. Let the current small node placement be $s \in S$ and assume small node $n$ is updating its location. For each small node in the mini-cells, it can jump up, down, left, right by one mini-cell, or remain in the original mini-cell. To be more concrete, consider the example in Fig. 2. Suppose that small node $n$ originally locates in mini-cell 5. In one update process, it can choose to move to mini-cell 8, mini-cell 2, mini-cell 4, or mini-cell 6. Or, it can simply remain at mini-cell 5. For the small node on the boundary of the area of interest, it can jump around the boundary, for example, mini-cell 9 can move to mini-cell 3, mini-cell 6, mini-cell 8, or mini-cell 7. Or, it can stay at mini-cell 9. Let $P(b_n|s)$ be the probability that small node $n$ updates its location to $b_n$. According to the Gibbs sampling [16], we can set these probabilities as

$$P(b_n|s) = \frac{e^{\gamma \mathcal{U}(b_n, s/n)}}{\sum_{(b_n', s/n)\in S} e^{\gamma \mathcal{U}(b_n', s/n)}}.$$  

(9)

Note that it is possible for $b_n = s_n$, for which case the small node $n$ simply remains in its original position.

### § Gibbs Sampling

1. **for** each iteration **do**
2. **for** each small node $n$ **do**
3. Tentatively move the small node up, down, left, right by one mini-cell, or remain in the original mini-cell
4. Calculate throughput for each user
5. Calculate transition probability according to (9)
6. Using the distribution (9), randomly choose the mini-cell that the small node $n$ will jump to
7. Update the node association for each user
8. **end** **for**
9. **end** **for**

**Lemma 1.** The expected utility generated by the Gibbs sampler in (9) is bounded from below by $\mathcal{U}(s^*) - \frac{1}{\gamma \log(S)}$.

**Proof:** Consider an optimal solution $s^* \in S^*$, we have

$$\mathbb{E}\{e^{\gamma (\mathcal{U}(s^*) - \mathcal{U}(s))}\} = \sum_{s \in S} e^{\gamma (\mathcal{U}(s^*) - \mathcal{U}(s))} \pi(s)$$

$$= \sum_{s \in S} \frac{\pi(s^*)}{\pi(s)} \frac{\pi(s)}{|S|} = \frac{|S|}{|S^*|} \pi(s^*)$$

(10)

where the last step is because $\pi(s^*) \leq \frac{1}{|S^*|}$. By Jensen’s inequality, we have

$$e^{\gamma (\mathcal{U}(s^*) - \mathbb{E}\{\mathcal{U}(s)\})} \leq \mathbb{E}\{e^{\gamma (\mathcal{U}(s^*) - \mathcal{U}(s))}\} \leq \frac{|S|}{|S^*|}.$$  

(11)

By taking logarithm on both sides, we have

$$\mathbb{E}\{\mathcal{U}(s)\} \geq \mathcal{U}(s^*) - \frac{1}{\gamma} \log\left(\frac{|S|}{|S^*|}\right).$$  

(12)

The implication of Lemma 1 is that, as $\gamma \to \infty$, the expected utility generated by the Gibbs sampler approaches the optimal value.

### V. Simulation

In our simulation, all the configurations about the macro nodes are from an existing 3G networks. We assume that each small node’s height is 3.5 meter and each user equipment’s height is 0.75 meter. Let $d_{n,i}$ denote the Euclidean distance between user $i$ and cell $n$. We assume that each small node is omni-directional, and the propagation gain between small node $n$ and user $i$ is given by

$$l_{n,i} = -30.7 - 38 \times \log_{10}(d_{n,i}).$$  

(13)

For macro node $n$, the propagation model follows the model in [13].

We run simulations over 3 data sets from an existing 3G network in a metropolitan area on the US western coast. Due to space constraints, we would briefly list out the results for data set 2 and data set 3 in Table. II and III, and only show in detail the results for data set 1. For data set 1, the area of interest has length $L = 700$ meters and width $W = 700$ meters. We evenly divide this area into 49 mini-cells. Thus, each mini-cell is exactly 100 meters long and 100 meters wide. In the area of interest,
the data traffic density for each mini-cell is also obtained from the existing 3G network. In Fig. 3, we show the relative data density value by a heat map.

A. Performance Comparison

We compare the performance of Gibbs sampling, greedy EcNo, and greedy hotspot in terms of the average throughput\(^5\) per user, the average throughput per cell, and the offloading factor. Among them, the offloading factor is defined as the ratio of number of users who have been offloaded from macro nodes to small nodes, to the number of all users in the area of interest. As we have explained in Section I, the offloading factor is important because as more users are offloaded, the burden on the macro nodes will be lower. Thus, the throughput and dropping rate for macro nodes can also be enhanced.

We divide the simulation into two stages: design and evaluation. In the design stage, we run different schemes in order to find a deployment decision for the small nodes. Then, in the evaluation stage, we run Monte Carlo simulations to measure the average cell/ user throughput, and the offloading factor.

In the design stage, in order to generate the deployment locations for small nodes, we first randomly generate 4900 users\(^6\), and place them in each mini-cell according to the data traffic density. To be more concrete, suppose that for mini-cell \(j\) the data traffic density is \(c_j\). Then the normalized traffic density for mini-cell \(j\) thus becomes \(\frac{c_j}{\sum_{k=1}^{N_a} c_k}\). Every user has a probability \(\frac{c_j}{\sum_{k=1}^{N_a} c_k}\) to be assigned to mini-cell \(j\). Once a user has been assigned to mini-cell \(j\), we uniformly generate a location within mini-cell \(j\) for this user. Then, we run the Gibbs sampling, greedy, and greedy hotspot algorithms individually, to obtain the deployment locations for all small nodes.

In the real world, seldomly as many as 4900 users would generate data traffic simultaneously. Thus, after producing the small node deployment results, in the evaluation stage we run the simulation with a small-number of users for calculating the throughput. We again generate 10 users randomly according to the data traffic density and calculate the throughput and offloading factor. By repeating the 10-user simulation for 1000 times, we measure the average throughput and the offloading factor.

\(^5\)We assume all the throughput in the simulation is counted as MB/Hour.

\(^6\)A large number of users help to capture the traffic demand information.

![Fig. 3](image-url) Illustration relative data traffic density in \(7 \times 7\) mini-cells. The darker the color in each mini-cell is, the heavier the traffic density is.

![Fig. 4](image-url) Map for the area of interest with small nodes (red triangle) and macro nodes (black circle)

Fig. 4 shows the geographical information of data set 1. The circles represent the 4 macro nodes. As we can see, the area of interest (a square) is far away from the 4 macro nodes and is on the boundary of the macro cells. The red markers inside the square represent locations of the small nodes produced by greedy EcNo algorithm and the numbers represent the node index. Fig. 5, 6, and 7 show the fixed locations of the small nodes. In Fig. 5 we also show 100 randomly generated users in order to give a sense of where they are. Note that the users are generated from the traffic density shown in Fig. 3, and are the same for Fig. 6, and 7 too. For each user, the number in Fig. 5 near it represents the node that the user associates with.

Due to the space constraints, we only discuss the deployment with 7 small nodes in the area of interest. In Figs. 5, 6, and 7, we show the deployment designs
for greedy EcNo, greedy hotspot, and Gibbs sampling algorithms. In Fig. 5, the greedy EcNo algorithm first places small node 5 in the center of the area, in order to greedily maximize the EcNo value of all users. All other nodes are deployed likewise. In Fig. 6, the greedy hotspot algorithm places all the small nodes on the hotspots, while ignoring all the mini-cells on the upper side of the map. In contrast, the Gibbs sampling approach jointly considers the user signal quality and geographic traffic information, and results in the decision in Fig. 7. In this deployment, near every hotspot we can find a small node, and all small nodes are spreaded out so that the interference is lower.

In Fig. 8, we show the sorted total throughput for all users in each of the 1000 instances of Monte Carlo simulation. We can see that in nearly 80% of the instances, the average throughput per user is larger than 1.4 MB/Hour. In Fig. 11 we see that with 7 small nodes Gibbs sampling increases the throughput by 4 times compared to the scenario without small nodes. Gibbs sampling also outperforms greedy approaches by 10%. In Fig. 12, we see that with 7 small nodes Gibbs sampling successfully offloaded on average almost all users to the small nodes. The offloading for Greedy hotspot is not as good, because its deployment ignores the areas that are not “hotspots”.

Fig. 5. Deployment map for greedy EcNo algorithm with 7 small nodes and 100 randomly generated users (each user is represented as a star)

Fig. 6. Deployment map for greedy hotspot algorithm with 7 small nodes

Fig. 7. Deployment map for Gibbs sampling algorithm with 7 small nodes

Fig. 8. Summation of user throughput in 1000 instances of Monte Carlo simulation of evaluation stage
B. The Convergence of Gibbs Sampling

The Gibbs sampling approach may take multiple iterations to converge. Hence, we would like to study how the Gibbs sampling approaches a steady state. In Fig. 9 we show the performance metrics in each iteration. Note that in Fig. 9 the global efficiency is defined as the average cell throughput divided by 3.84 (i.e., the effective bandwidth for HSDPA, which is 3.84MHz). We can see that the performance metrics get stable after 15 iterations. Note that the performance have some oscillations because there exist other deployments with close-value of user throughput.

C. Number of small Nodes

In this part, we study how the number of small nodes would affect the throughput. As we see from Fig. 11, the throughput per user increases as the number of small nodes increases. The Gibbs sampling approach outperforms greedy and greedy hotspot approach across different number of small nodes. Although the average throughput of greedy hotspot is not far from Gibbs sampling in Fig. 11, from Fig. 12 we see that the offloading factor for greedy hotspot is much poorer. It’s easy to imagine this kind of result, especially for the area which contains some isolated hotspots. In some cases, if the number of small nodes is low, users far away from the hotspot would not be offloaded to the small nodes.

VI. CONCLUSION

The growing popularity of mobile data services have placed greater demands for cellular networks to support higher throughput. To support the rapidly growing traffic demand, cellular providers are considering to deploy small nodes to supplement the existing network. In this

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**Table II**

<table>
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<td>avg user thpt offloading</td>
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**Table III**

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**Fig. 9.** Average user throughput and global efficiency in each iteration of Gibbs sampling

**Fig. 10.** Average cell throughput and average CQI values for cells in each iteration of Gibbs sampling

**Fig. 11.** The average user throughput with different number of small nodes
paper, we study the optimization for deployment of small nodes in 3G networks. To our best knowledge, this work is the first solution for the placement of small-cell base-stations that jointly optimizes the locations of multiple small nodes with the goal of maximizing any given network utility function. Specifically, we focus on an area of interest on the boundary of macro cell. All macro nodes are far away from the area of interest. We first present greedy algorithms with low complexity. However, their performances can be insatisfactory. We then propose to use Gibbs sampling to optimize the small node deployments. The Gibbs Sampling method intelligently balances two potentially conflicting considerations of placing small-cell BSs close to hotspots and avoiding interference with the macro-cell BSs & other small cell BSs. We show that it converges to the deployment decision with the maximum total system throughput with high probability. We run simulations using the setting from an existing 3G network, and show that Gibbs sampling can produce better throughput performance than greedy algorithms.

For future work, we plan to refine the proposed algorithm by taking into account the effect of additional interference management mechanisms. For example, neighbor cells can appropriately schedule their transmission in the time domain to avoid co-channel interference. Further, they adjust their transmission power to avoid excessive interference. It would be interesting to see how the deployment decisions should be optimized jointly with the scheduling and power control mechanisms.

Fig. 12. The offloading factor with different number of small nodes

REFERENCES