* The connection between DTFT & the Z transform.

Analysis formula

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\left(e^{j\omega}\right)^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n](z)^{-n}$$

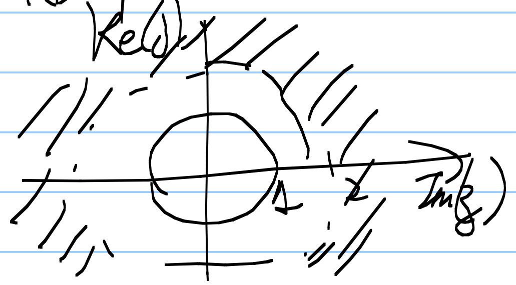
* Implication 1: If we know the expression of $X(z)$ & its ROC.

And If the ROC also contains $e^{j\omega} = z$, the unit circle,

then we can evaluate the DTFT by pluging in $z = e^{j\omega}$

* Also, the DTFT exists if the ROC of $X(z)$ includes the unit circle.

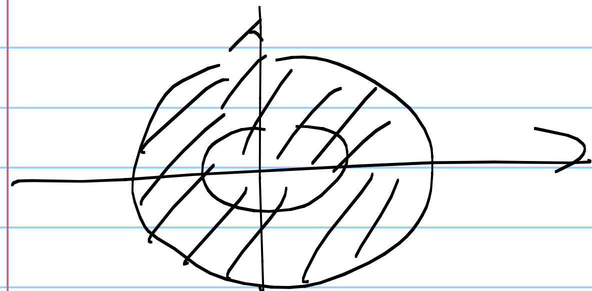
Example: $x[n] = 2^n u[n]$



the ROC does not contain the unit circle.

$\Rightarrow$

$x[n] = -2^n u[-n-1]$



$\Rightarrow$ ROC contains the unit circle

& $X(z) = \dfrac{1}{1-2z^{-1}}$

$\Rightarrow$

* It is critical to check whether the ROC contains the unit circle before using the z-transform to derive the DTFT

* Inverse z transform: Using the inverse DTFT to find out the inverse z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n](z)^{-n}$$

Detailed steps:

Step 1: Find an arbitrary r value such that $|z|=r$ circle is in the ROC. Fix that r.

Step 2: Use $X(z)$ to construct

Step 3: Find the Inverse DTFT of $Y(e^{j\omega})$ ($y[n]$)

Step 4:

Example: $X(z) = \dfrac{1}{1-2z^{-1}}$, ROC: $|z|>2$. Find $x[n]$.

Ans: Solution 1: Table look up.
p.775, 776 Table 10.2

Solution #2:

Step 1:

Step 2:

Step 3:

Step 4:

Example: $X(z) = \frac{1}{1-2z^{-1}}$, ROC: $|z| < 2$. Find $x[n]$.

Ans: Step 1:

Step 2:

Step 3: Find $y[n]$.

DTFT Table look up ?

This technique can also be used to solve Spring 08 MB Q3.