p.15 ✡ In many cases, $X(z)$ does not exist.

Note Title 4/23/2010

⟹ The concept of [ ]

✡✡ $X(z)$ exists only when $\sum_{n=-\infty}^{\infty} x[n] z^{-n}$ converges. ≡ only when $|z|$ is within some range.

Ex: If $x[n] = 2^n u[n]$

✡ The set of values of z (z is in the complex plane) is called the "Region of convergence" (ROC) of the Z — Transform

Example: The Z transform of $x[n] = 2^n u[n]$ & its corresponding ROC.

Ans:

Note: The Z transform expression outside its ROC is meaningless. (Not even exist.)

We should say

$X(z) =$

$\Rightarrow$ Very clear. But too long.

We engineers say that

$X(z)$ is [ ] & the Region of Convergence (ROC) is [ ] $\Rightarrow$ shorter, but you need to be careful

Example: $x[n] = -2^n u[-n-1]$

Find the Z Transform & its ROC

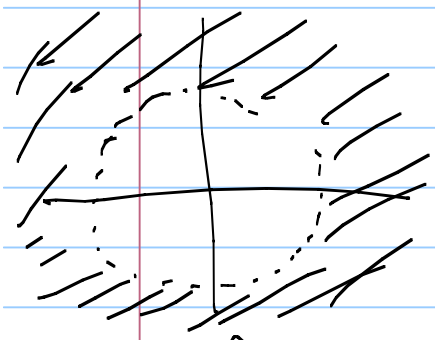
Ans:

If we use the long expression

I

$$X(z) = \begin{cases} \\ \\ \\ \end{cases}$$

then it is clear that the new $X(z)$ is different from that of the previous example.



II

$$X(z) = \begin{cases} \frac{1}{1-2z^{-1}} & \text{if } z \text{ is outside the } (1/2) \text{ circle} \\ \text{does not exist} & \text{if } z \text{ is inside the } (1/2) \end{cases}$$

But if we use the shorter expressions:

I $X(z) = \frac{1}{1-2z^{-1}}$ & the ROC is inside $(1/2)$

II $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$ & the ROC is outside $(1/2)$

* Same $X(z)$ expression as in the previous example, but different ROC. confusing

* ROC is an integral part of ZT.

*

(even though the expressions may be the same). This is clear from the long expression.

* When asked finding the ZT, we need to specify both the expression & the ROC.

HW14Q118 Prob 10.22(b)

$x[n] = n\left(\frac{1}{2}\right)^{|n|}$

Q: Find $X(z)$

Ans:

Long expression

Short expression