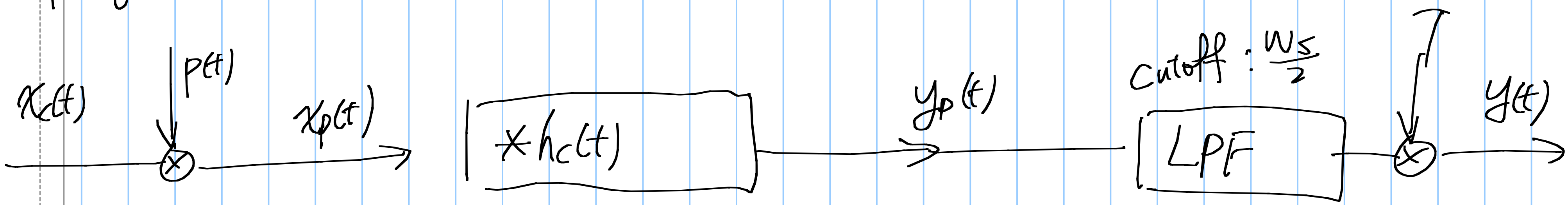
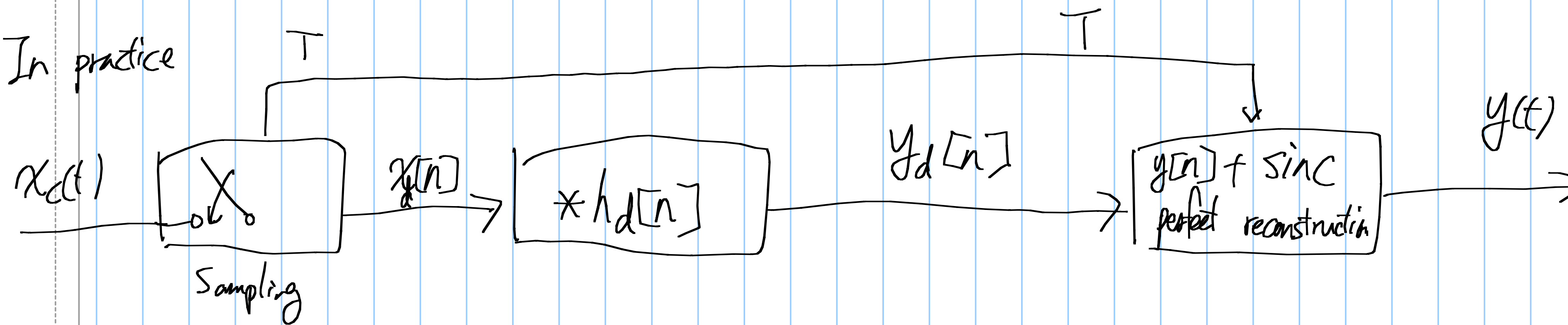
Note Title 4/25/2014

Digital Signal Processing

Conceptually



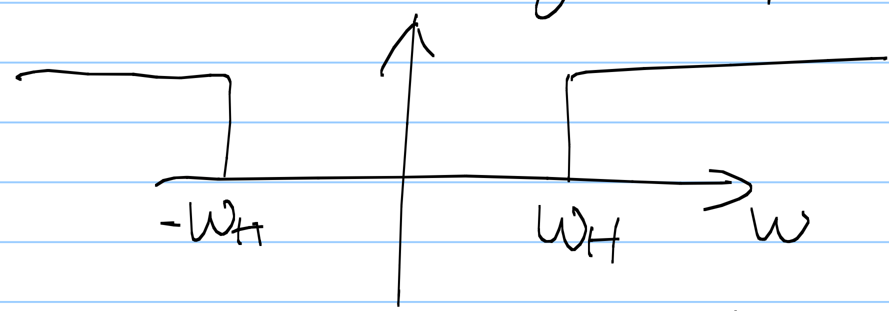
In practice



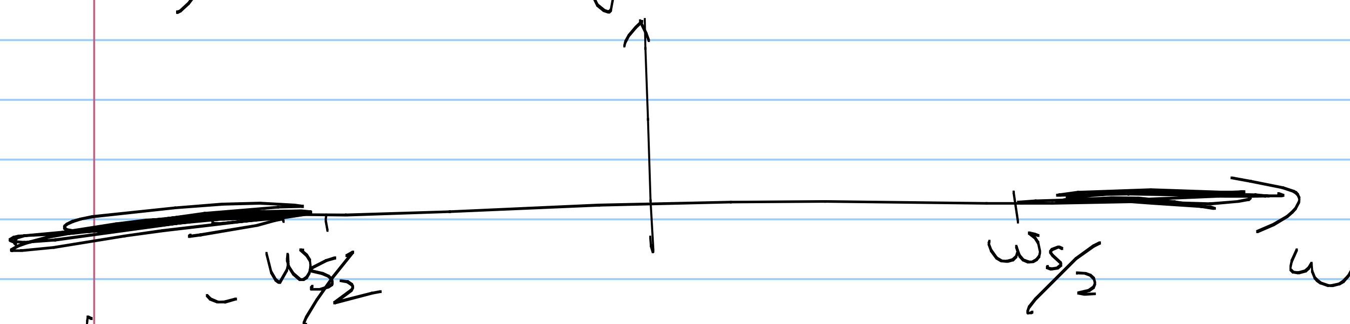
Goal: the input/output relationship between $x(t)$ & $y(t)$ fits the given design goal $h(t)$. or equivalent $H(j\omega)$

Observation #1:

Suppose our goal is to design a HPF with freq response



Because the LPF used in reconstruction there will be "zero" freq component beyond the range



$\Rightarrow$ Our new, modified goal becomes

$x_c(t)$: the original Conti signal
$x_d[n]$: The sampled array. (DT signal)
$x_p(t)$: the impulse train sampling

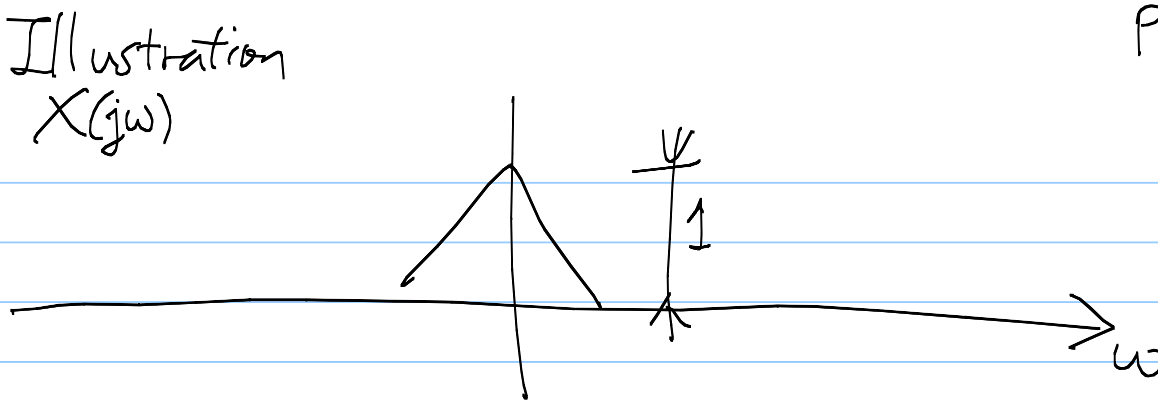
* Observation #2

Let's first take the DTFT of $x_d[n]$ & Compare it with the CTFT of $x_p(t)$.

versus

More explicitly $X_d(e^{jw})$ will have the same amplitude & shape of $X_p(jw)$ except now $X_d(e^{jw})$ is of period 2π while $X_p(jw)$ is of period w_s ← $\boxed{\text{Observation \#2}}$

Illustration



Remark: the Bandlimited reconstruction does the inverse: Stretch $(0 \Leftrightarrow 2\pi)$ back to $(0, \omega_s)$ & remove the side copies.

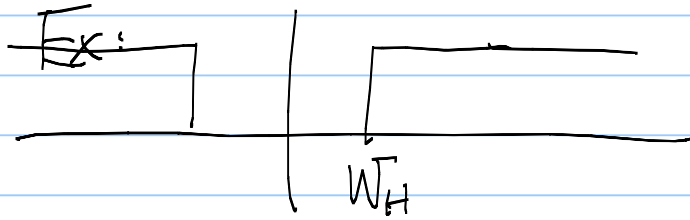
Observation #3

We would like to maintain the mirroring relationship between the conceptual ITS system & the practical discrete-time signal processing sys.

Therefore, if we can make sure that $H_c(j\omega)$ has the same freq spectrum as $H_{step1}(j\omega)$, then the final output will fit the desired $H(j\omega)$. (At least in the $(-\frac{\omega_s}{2}, \frac{\omega_s}{2})$ region.)

p.203 Given the desired $H(j\omega)$, the best we can do is

Step 1: Chop-off freq response outside $\left(-\frac{\omega_s}{2}, \frac{\omega_s}{2}\right)$



$H(j\omega)$

Step 2: However, in practice, we can only design $h_d[n]$. Therefore, we stretch $H_{step1}(j\omega)$ to fit the $(-\pi, \pi)$ period

$H_{step1}(j\omega)$

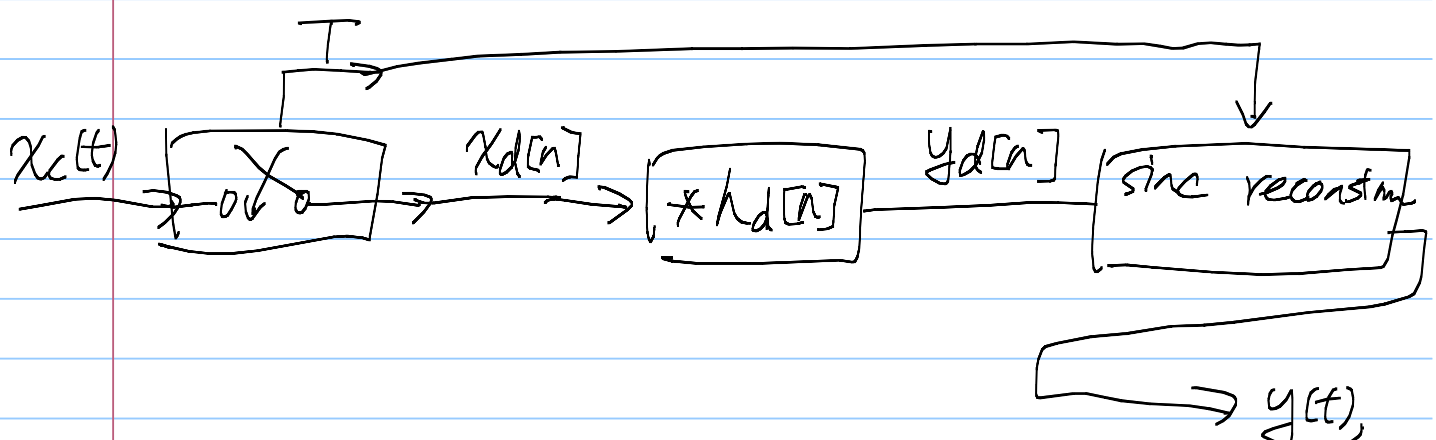
Therefore, if we can make sure $H_d(e^{j\omega})$ has the same spectrum as $H_{step2}(e^{j\omega})$, then the $Y_d(e^{j\omega})$ will be a stretched vesion of $Y_p(e^{j\omega})$.

We thus maintain the mirror relationship betwe the conceptual & the practical systems.

$\Rightarrow$ The output of the practical system also fits the desired freq response $H(j\omega)$.

Step 3:

The end result becomes



which has the desired end to end $H(j\omega)/h(t)$.