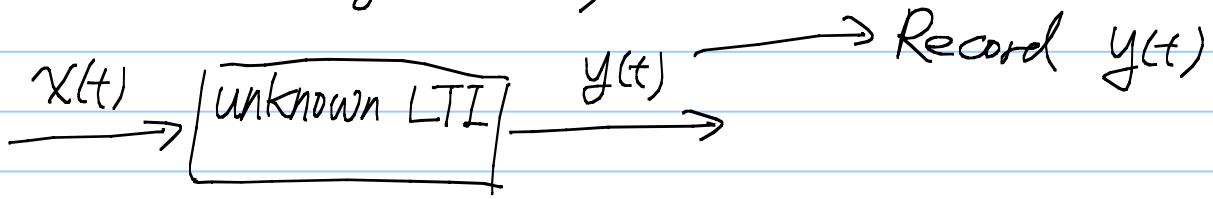
✳ Other Applications of the convolution property:

1. Characterizing / Identifying LTI systems

Recall: In the past, we record h(t) by sending input $\delta(t)$. Then we compute $H(j\omega)$

An alternative way is to find $H(j\omega)$ directly

Namely, arbitrarily choose $x(t)$ and send it through the sys.




Find $X(j\omega)$ & $Y(j\omega)$ by F.T, which can be done by computer.

$$\because Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$\therefore H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

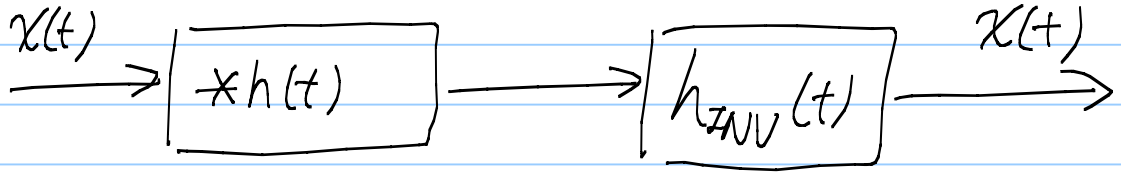
$$\Rightarrow h(t) = \mathcal{F}^{-1}(H(j\omega))$$

⇒ We do not need to feed an impuse signal to an LTI system.

Note that an "impulse" is very hard to generate since it has $\infty$ amplitude.

## 2. Inverting LTI sys.

Recall: If a sys is invertible




⇒ The impulse response of the concatenated system is

Q: $\mathcal{F}(\delta(t)) = ?$

Ans:

In summary, given $h(t)$, the $h_{INV}(t)$ can be found as follows.

Example: $h(t) = e^{-t}u(t)$, find the inverse system

Soln:

* An LTI sys. is invertible if $H(j\omega) \neq 0$ for all $\omega$.

Note Title 3/4/2012

2. Using "convolution in time $\equiv$ multiplication in freq" to invert an LTI system.

Example: Is $y(t) = \int_{t-T_1}^{t+T_1} x(s)\,ds$ invertible?

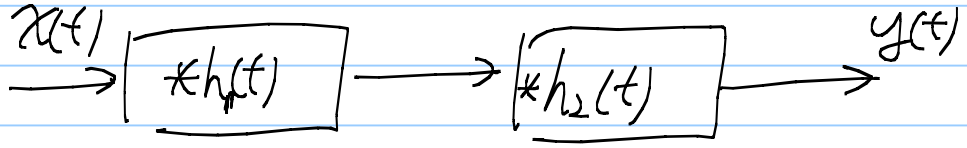
Ans:

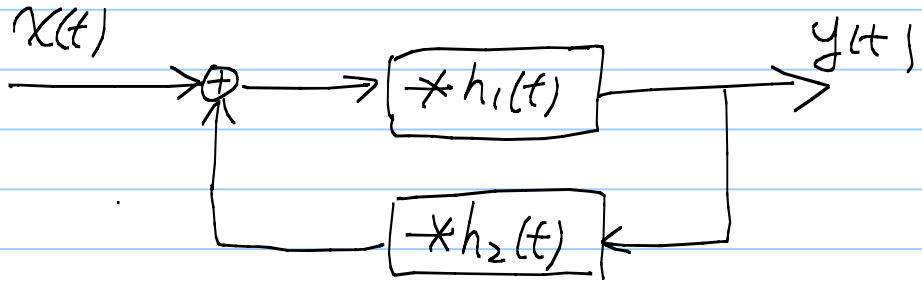
Example: $h(t) = \dfrac{\sin(Wt)}{\pi t}$ for some $W > 0$

Is this system invertible?

Ans:

3. Interconnection of LTI sys.
(similar to Laplace Transform)




Like you have learned in Laplace transform

4. Freq-based manipulation of the signal ✡✡✡

Example:
An ideal low-pass filter (LPF) will ① keep any freq component within $|\omega| < W$ intact (also known as the bandwidth of a LPF)
② Completely supress any freq component outside (i.e $|\omega| > W$)

Since the output is

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$\Rightarrow H_{Ideal,LPF}(j\omega)$$

So if we can design an LTI system with $h(t)$ satisfying $H(j\omega) = H_{Ideal,LPF}(\omega)$, then we have the ideal LPF

$\Rightarrow h(t) =$