

* Properties of CTFT. (Sec 4.3 of the textbook.)

① Linearity

$$ax(t) + by(t) \xleftrightarrow{\text{F.T}}$$

② Time-shift

$$x(t) \xleftrightarrow{\text{F.T}} X(j\omega)$$

$$y(t) = x(t - t_0) \longleftrightarrow$$

pf:

③ Freq-shift

$$x(t) \xleftrightarrow{\text{F.T}} X(j\omega)$$

$$y(t) = \longleftrightarrow Y(j(\omega - \omega_0))$$

④ Time-Reversal \equiv Freq Reversal

$$y(t) = x(-t) \longleftrightarrow$$

⑤ Time-scaling: for some $a > 0$

$$y(t) = x(at) \longleftrightarrow$$

* An example about the freq-shift property.

Example: $X(j\omega) = U(\omega+3) - U(\omega-3)$

P.12

(HW9Q78): Find $x(t)$ & Plot it.

Ans:

$$z(t) = x(t) e^{j3t} \quad \text{Q: Find the}$$

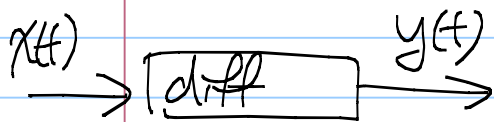
F.T of $z(t)$.

Ans:

⑥ Differentiation

$$y(t) = \frac{dx(t)}{dt} \quad \longleftrightarrow$$

pf:



⇒ Differentiation is a high-pass filter

∴ High freq components are amplified

by $|j\omega| = |\omega|$

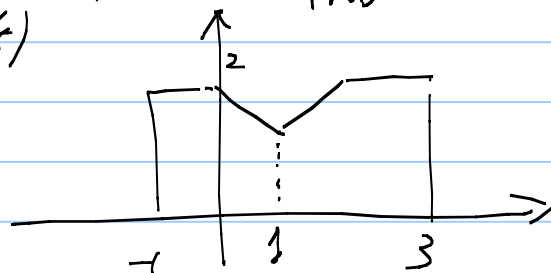
Slow movement $\xrightarrow{\text{diff}}$ Small values

fast movement \rightarrow large values

⑦ Parseval's Relationship (Law of energy conservation)

Example:

Prob 4.25 (a, b, c, e)

 $X(t)$ (a) Find $\mathcal{F}\{X(t)\}$

Ans:

(b) Find $X(j\omega)$

Ans:

(c) Find $\int_{-\infty}^{\infty} X(j\omega) d\omega$

Ans:

(e) Find $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

Ans: