(3) Time reversal

$x(t) \xleftrightarrow{F.S} a_k, \omega_0$

$y(t) = x(-t)$

$y(t) \xleftrightarrow{F.S} b_k,$ □

Ans:

pf 1: Direct computation — p.203

pf 2: Inspection

④ Time scaling

$x(t) \longleftrightarrow a_k, \omega_0$

$y(t) = x(\alpha t) \quad \alpha > 0$.

$y(t) \longleftrightarrow b_k,$ □

Ans:

pf 1: Direct computation — p.204

pf 2: By inspection

Remark 1: Time-scaling is the only property that involves freq-change

Remark 2: Two F.S. representations are the same only when

$$a_k = b_k \quad \text{and} \quad \underbrace{\omega_1 = \omega_2}_{\text{the freq}}$$

**Question for the teams**

Prove that "If $x(t)$ is an even signal, then $a_k = a_{-k}$ for all $k$."

Prove that "if $a_k = a_{-k}$ for all $k$, then $x(t)$ is an even signal".

Can you derive similar arguments for the case in which $x(t)$ is an odd signal?