* Convergence of the Fourier Series.

Q: Do all CT periodic signals x(t) have a Fourier series representation, i.e. can every periodic signal be written as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \ ?$$

A: The answer is no. However, we can get close in most cases.

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* We can find the Fourier series representation for the following types of signals:

(1) Any "continous" signal x(t) that has no abrupt changes can be expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

(2) If a "mostly continuous" signal x(t) does not go to infinity and has a finite number of abrupt points in a single period T, then only at those points, we have

$$x(t) \neq \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

For all those "continuous" regions, the FS representation holds.

* See Conditions 1 to 3 in P. 197 of the textbook for detailed discussion.

* Properties of the CT FS.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{+jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt$$

That is

$$x(t) \xleftrightarrow{FS} (a_k, \omega_0)$$

time-domain representation — freq-domain representation

Given $x(t)$, we find a_k by ① inspection or ② direct computation.

Suppose $x_2(t) = x(t-t_0)$ is a shifted version of the original $x(t)$ & we have spent a lot of time to compute a_k

$$x(t) \xrightarrow[\text{② direct computation}]{\text{① inspection}} a_k, \omega_0$$

$$x_2(t) = x(t-t_0) \longleftrightarrow$$

Q: Can we directly compute b_k from a_k w/o reapplying ① or ②

A:

* Properties of FS. (see Table 3.1 for a complete list of properties.

① Lineariety: Suppose both $x(t)$ & $y(t)$ have period T.

$x(t) \longleftrightarrow a_k,$

$y(t) \longleftrightarrow b_k,$

$z(t) = Ax(t) + By(t)$

$z(t) \longleftrightarrow c_k,$

pf:

Note Title 2/12/2010

② Time-Shift property

$$x(t) \xleftrightarrow{F.S} (a_k, \omega_0)$$

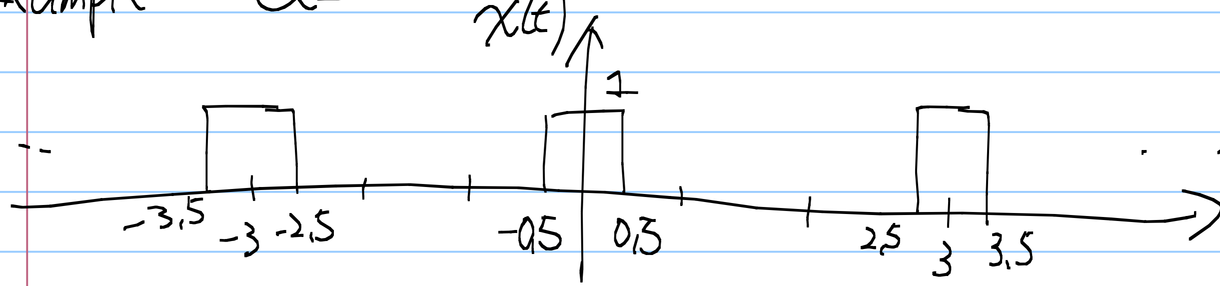
$$y(t)=x(t-t_0) \xleftrightarrow{F.S} (b_k, \boxed{})$$

Ans :

pf ①: Direct computation — P.202

②: By inspection

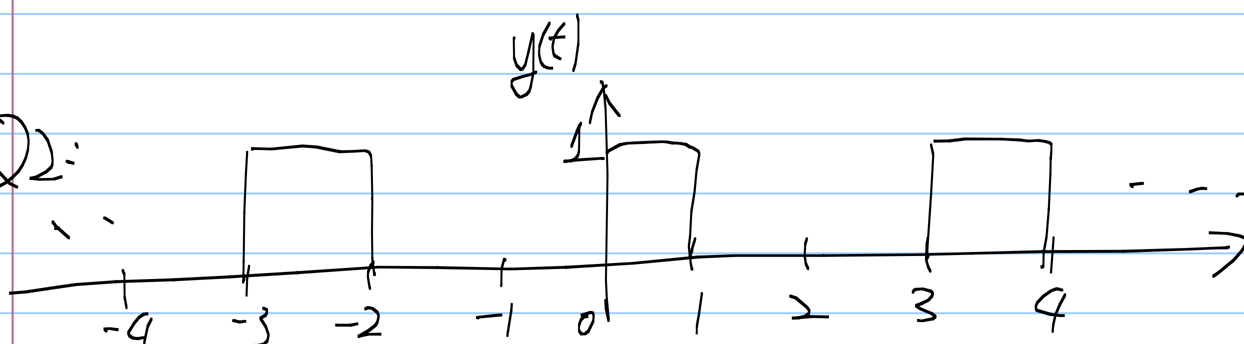
Example Q1



Find the FS of $x(t)$

Ans:

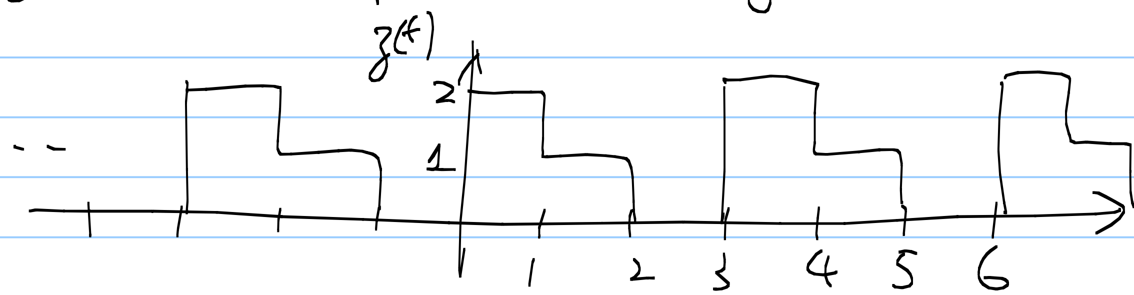
Q2:



Find the FS of $y(t)$

Ans:

Q3: HW6Q57 Prob 3.22(a) fig (f)



Find the FS of $z(t)$

Ans: