* Alternative way of finding the inverse Z-transform.

① Inspection:

Example 10.12

$X(z) = 4z^2 + 2 + 3z^{-1}$ for $0 < |z| < \infty$

Find $x[n]$

Ans: $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$\Rightarrow x[-2] = 4 \qquad x[1] = 3$

$x[0] = 2$

& all other $x[n] = 0$.

② Inspection plus power series.

$X(z) = e^z$ for $|z| < 1$

Find $x[n]$

Ans: By Taylor's expansion,

if $|z|<1$, then

$$e^z = f(z) = f(0) + \frac{f(1)}{1!}(z-0) + \frac{f(2)}{2!}(z-0)^2$$

$$= \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{0!} z^k$$

since $f(z)=e^z$. $f^{(k)}=1$ for all k

$$\Rightarrow e^z = 1 + \frac{1}{1} z + \frac{1}{2!} z^2 + \frac{1}{3!} z^3 + \cdots$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$\Rightarrow x[0]=1$$

$x[-k] = \frac{1}{k!}$ for $k>0$

$x[k]=0$ for $k>0$

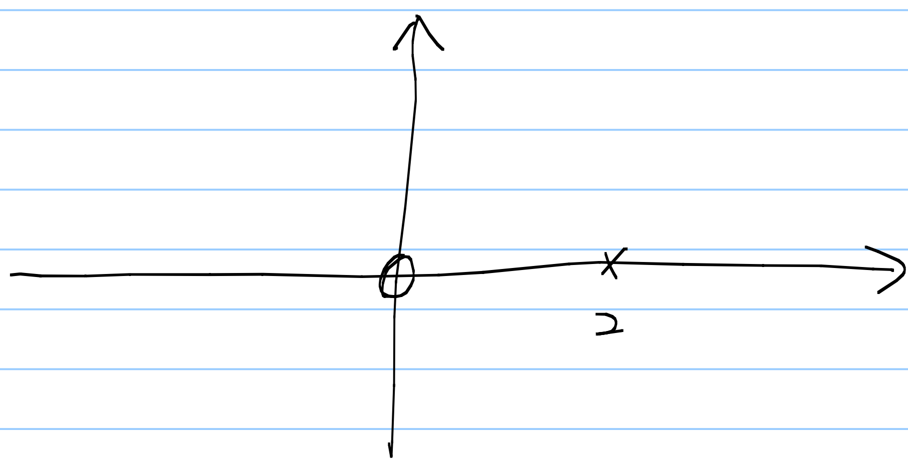
Exercise Example 10.13. 10.14

* To discuss the ROC of a Z transform, we need the $\boxed{\text{pole-zero plot}}$ of a Z-transform

poles: the z values for which the denominator is zero, "x"

zeros: the z values for which the numerator is zero. "o"

Example: $X(z) = \dfrac{1}{1-2z^{-1}}$



* Make sure to check $z=0$ & ∞.

When $z=0$ $X(z) = \dfrac{1}{\infty} = 0$

$\Rightarrow$ $z=0$ is also a zero.

When $z=\infty$ $X(z) = \dfrac{1}{1}$, neither a pole, nor a zero.

* When computing the poles & zeros, we temporarily ignore the ROC.

One of the very important properties of Z-transform:

$$y[n] = x[n] * h[n]$$

$$Y(z) = X(z) \cdot H(z)$$

& the ROC of Y contains the intersection of two ROCs. (joint ROCs)