p.215 In many cases, $X(z)$ does not exist.

$\Rightarrow$ The concept of $\boxed{\text{Region of Convergence}}$

$X(z)$ exists only when $\sum_{n=-\infty}^{\infty} x[n] z^{-n}$ converges. $\equiv$ only when $|z|$ is within some range.

Ex: If $x[n] = 2^n u[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} 2^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (2 \cdot z^{-1})^n$$

exists only when $|2 \cdot z^{-1}| < 1 \Leftrightarrow |z| > 2$

& the final expression is

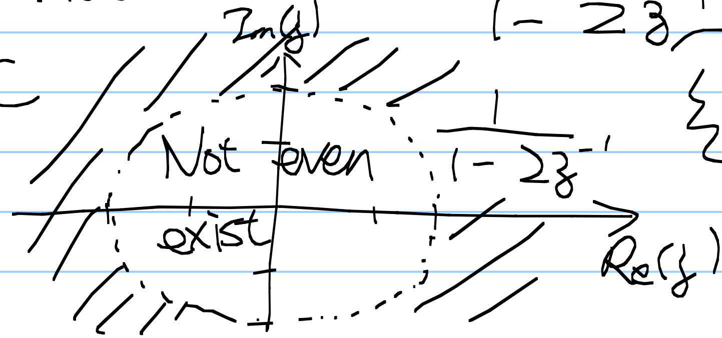
$$\frac{1}{1-2z^{-1}} \quad \#$$

The set of values of z (z is in the complex plane) is called the "Region of convergence" (ROC) of the Z-Transform

Example: The Z transform of $x[n] = 2^n u[n]$ & its corresponding ROC.

Ans: $X(z) = \dfrac{1}{1-2z^{-1}}$ with a

ROC



the circle is not included

Note: The Z transform expression outside its ROC is meaningless.

(Not even exist.)

We should say

$$X(z) = \begin{cases} \dfrac{1}{1-2z^{-1}} & \text{if } z \text{ is outside the } \left(\tfrac{}{2}\right) \text{ circle} \\ \text{does not exist} & \text{if } z \text{ is inside the } \left(\tfrac{}{2}\right) \end{cases}$$

$\Rightarrow$ Very clear. But too long.

We engineers say that

$X(z)$ is $\dfrac{1}{1-2z^{-1}}$ & the Region of Convergence (ROC) is "outside (2)" $\Longrightarrow$ shorter, but you need to be careful

Example: $x[n] = -2^n u[-n-1]$

Find the Z Transform & its ROC

Ans: $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$$= \sum_{n=-\infty}^{-1} -2^n z^{-n} = \sum_{n'=1}^{\infty} -2^{-n'} z^{n'}$$

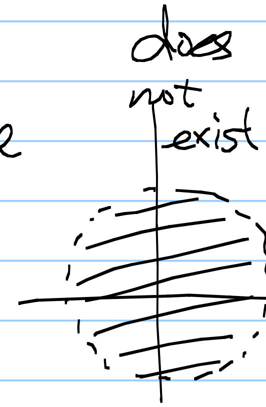
if $|2^{-1} \cdot z| < 1$
$\Leftrightarrow |z| < 2$

$$= \frac{-2^{-1} z}{1 - 2^{-1} \cdot z} = \frac{1}{1-2z^{-1}}$$

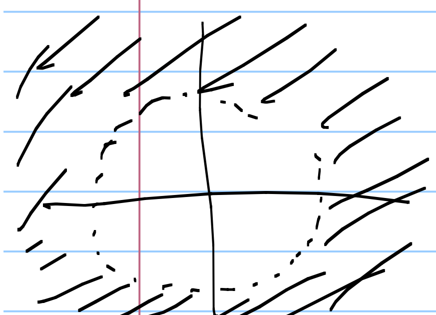
If we use the long expression

$\boxed{\text{I}}$

$$X(z) = \begin{cases} \text{does not exist} & \text{if } z \text{ is outside the } \left(\tfrac{1}{2}\right) \text{ circle} \\ \dfrac{1}{1-2z^{-1}} & \text{if } z \text{ is inside the } \left(\tfrac{1}{2}\right) \end{cases}$$



then it is clear that the new $X(z)$ is different from that of the previous example.



$\boxed{\text{II}}$

$$X(z) = \begin{cases} \dfrac{1}{1-2z^{-1}} & \text{if } z \text{ is outside the } \left(\tfrac{1}{2}\right) \text{ circle} \\ \text{does not exist} & \text{if } z \text{ is inside the } \left(\tfrac{1}{2}\right) \end{cases}$$

But if we use the shorter expressions:

$\boxed{\text{I}}$ $X(z) = \dfrac{1}{1-2z^{-1}}$ & the ROC is inside $\left(\tfrac{1}{2}\right)$

$\boxed{\text{II}}$ $X(z) = \dfrac{1}{1-2z^{-1}}$ & the ROC is outside $\left(\tfrac{1}{2}\right)$

* Same $X(z)$ expression as in the confusing previous example, but different ROC.

* ROC is an integral part of ZT.

* Different ROCs $\Rightarrow$ different Z-transform

(even though the expressions may be the same). This is clear from the long expression.

* When asked finding the ZT, we need to specify both the expression & the ROC.

HW14Q5 Prob 10.22(b)

$x[n] = n\left(\frac{1}{2}\right)^{|n|}$

Q: Find $X(z)$

Ans: $X(z) = \sum_{n=-\infty}^{\infty} n\left(\frac{1}{2}\right)^{|n|} z^{-n}$

$= \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n} z^{-n} + \sum_{n=-\infty}^{-1} n\left(\frac{1}{2}\right)^{-n} \cdot z^{-n}$

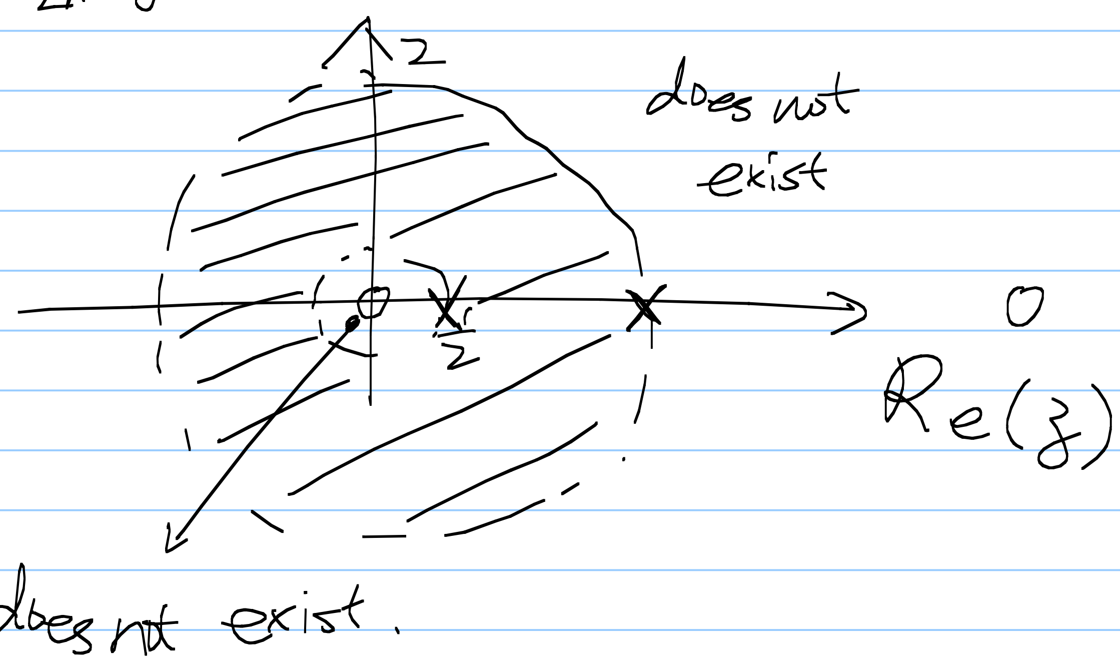
$= \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n} z^{-n} + \sum_{n'=1}^{\infty} -n'\left(\frac{1}{2}\right)^{n'} \cdot z^{n'}$

$= \dfrac{\frac{1}{2} z^{-1}}{\left(1-\frac{1}{2} z^{-1}\right)^2} + \dfrac{-\frac{1}{2} z}{\left(1-\frac{1}{2} z\right)^2}$

if $\left|\frac{1}{2} z^{-1}\right| < 1$ & $\left|\frac{1}{2} z\right| < 1 \iff \frac{1}{2} < |z| < 2$

Long expression

$$\Rightarrow X(z) = \begin{cases} \dfrac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2} + \dfrac{-\frac{1}{2}z}{(1-\frac{1}{2}z)^2} & \text{if } \frac{1}{2} < |z| < 2. \\ \text{does not exist} & \text{if otherwise} \end{cases}$$



Short expression

$$X(z) = \frac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2} - \frac{\frac{1}{2}z}{(1-\frac{1}{2}z)^2}$$ & the

ROC is $\frac{1}{2} < |z| < 2$.