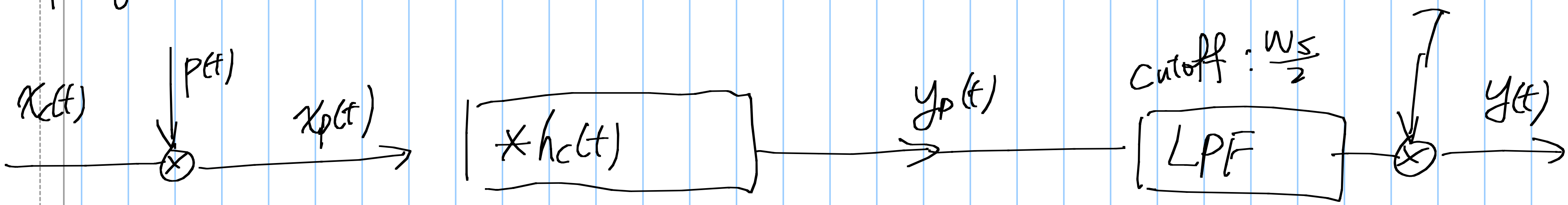
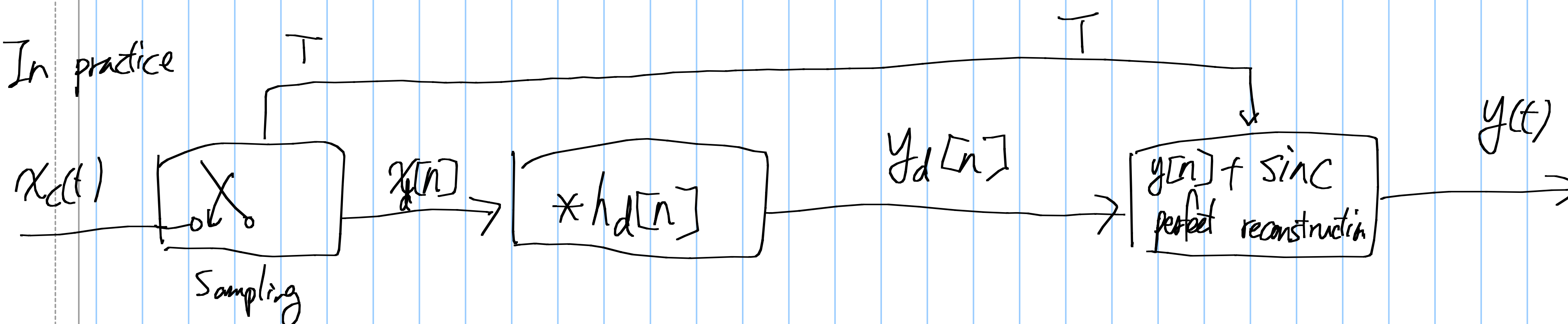
Digital Signal Processing

Conceptually



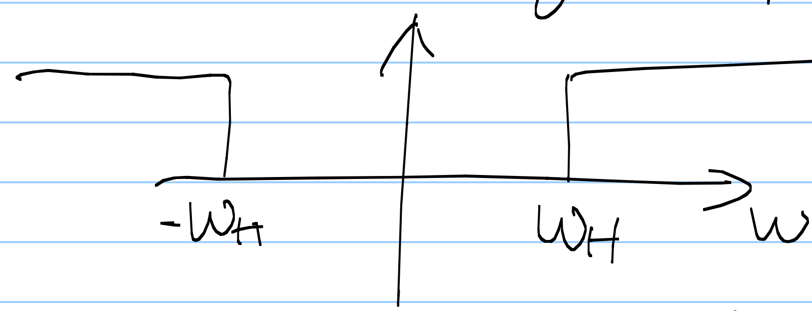
In practice



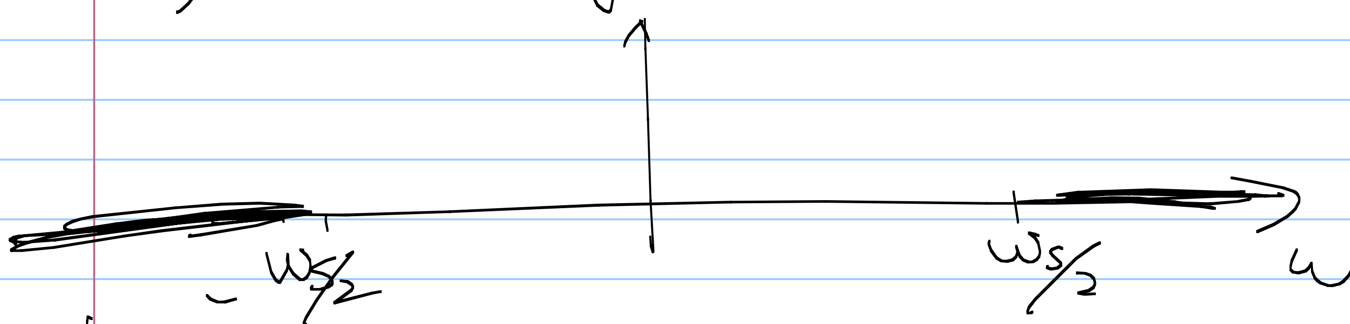
Goal: the input/output relationship between $x(t)$ & $y(t)$ fits the given design goal $h(t)$. or equivalent $H(j\omega)$

Observation #1:

Suppose our goal is to design a HPF with freq response



Because the LPF used in reconstruction there will be "zero" freq component beyond the range



⇒ Our new, modified goal becomes to design a Band-pass filter

$x_c(t)$: the original Conti signal
$x_d[n]$: The sampled array, (DT signal)
$x_p(t)$: the impulse train sampling

* Observation #2

Let's first take the DTFT of $x_d[n]$ & Compare it with the CTFT of $x_p(t)$.

$$X_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_d[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-j\omega \underline{\underline{n}}}$$

Note: $X_d(e^{j\omega})$ has period 2π

versus

$$x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\, \delta(t - nT)$$

$$X_p(j\omega) = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-j\omega \underline{\underline{nT}}}$$

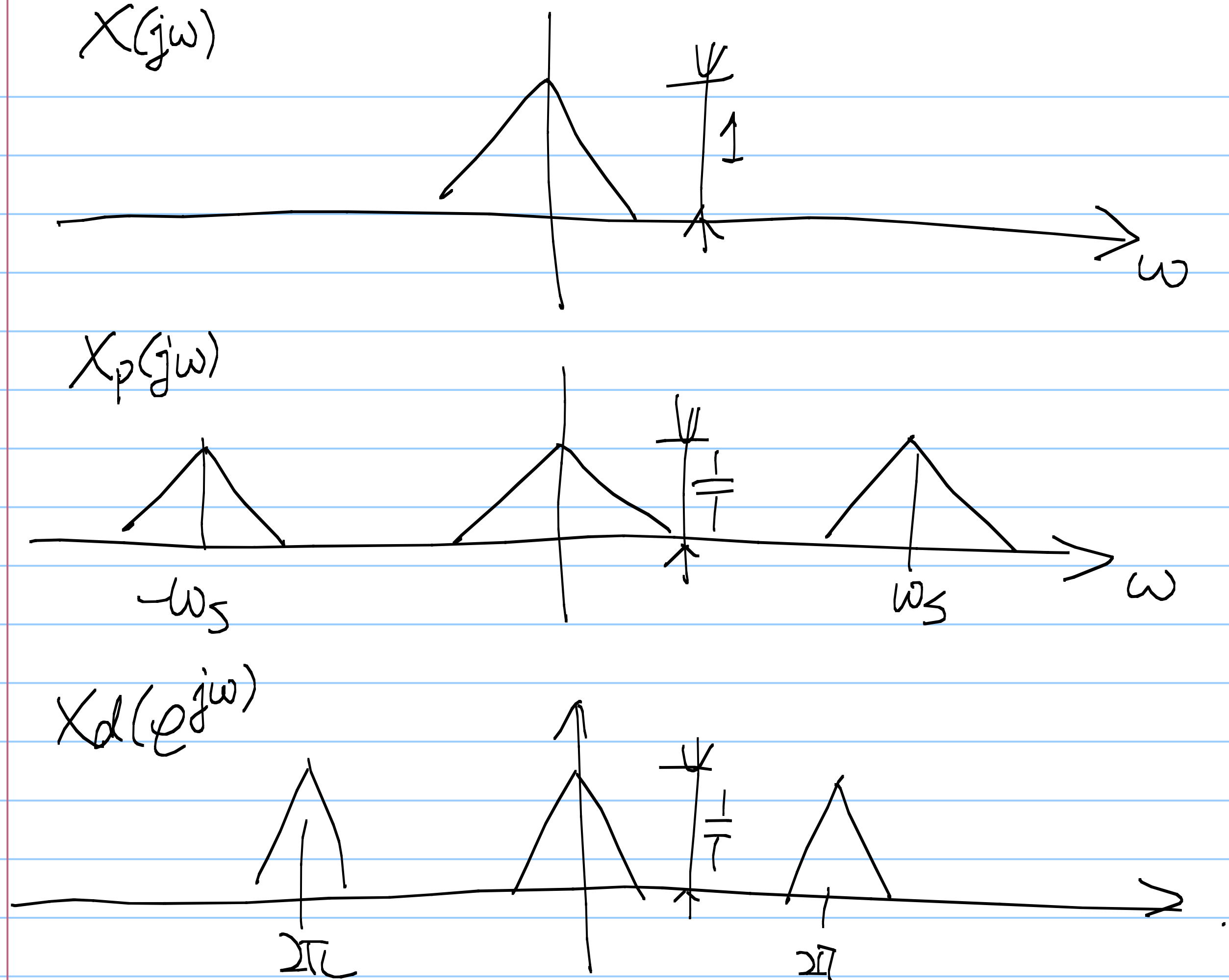
Note: $X_p(j\omega)$ has period ω_s

$\Rightarrow X_p(j\omega)$ is the freq-stretched version of $X_d(e^{j\omega})$ by a factor of T.

$$\boxed{X_d(e^{j\omega}) = X_p\left(j\frac{\omega}{T}\right)}$$

More explicitly $X_d(e^{j\omega})$ will have the same amplitude & shape of $X_p(j\omega)$ except now $X_d(e^{j\omega})$ is of period 2π while $X_p(j\omega)$ is of period ω_s $\leftarrow$ $\boxed{\text{Observation \#2}}$

Illustration



Remark: the Bandlimited reconstruction does the inverse: Stretch $(0 \Leftrightarrow 2\pi)$ back to $(0, \omega_s)$ & remove the side copies.

Observation #3

We would like to maintain the mirroring relationship between the conceptual CTS system & the practical discrete-time signal processing sys.

Therefore, if we can make sure that $H_c(j\omega)$ has the same freq spectrum as $H_{step1}(j\omega)$, then the final output will fit the desired $H(j\omega)$. (At least in the $(-\frac{\omega_s}{2}, \frac{\omega_s}{2})$ region.)

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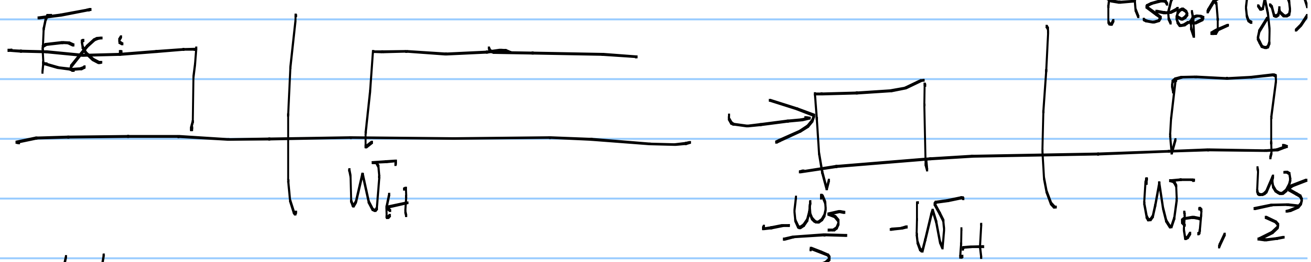
Given the desired $H(j\omega)$, the best we can do is

Step 1: Chop-off freq response outside $\left(-\frac{\omega_s}{2}, \frac{\omega_s}{2}\right)$ $\Downarrow$ $H_{step1}(j\omega)$

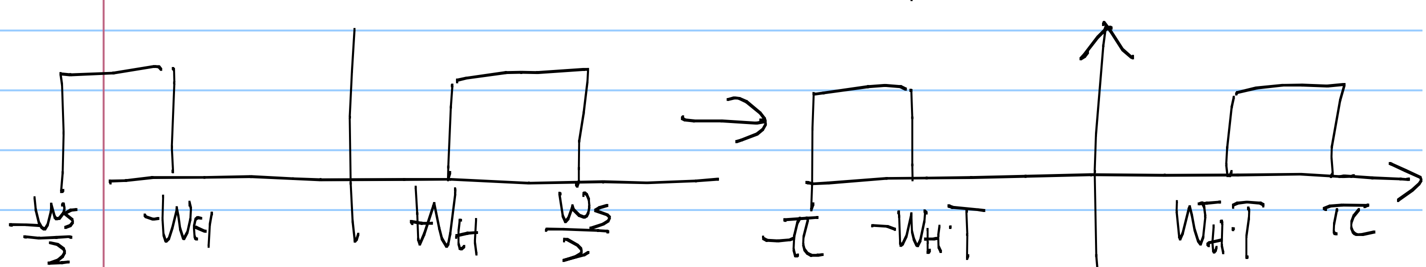
~~Ex:~~



$H(j\omega)$ $\quad\quad\quad\quad$ $H_{step1}(j\omega)$

Step 2: However, in practice, we can only design $h_d[n]$. Therefore, we stretch $H_{step1}(j\omega)$ to fit the $(-\pi, \pi)$ period

$H_{step1}(j\omega)$ $\quad\quad\quad\quad$ $H_{step2}(e^{j\omega})$



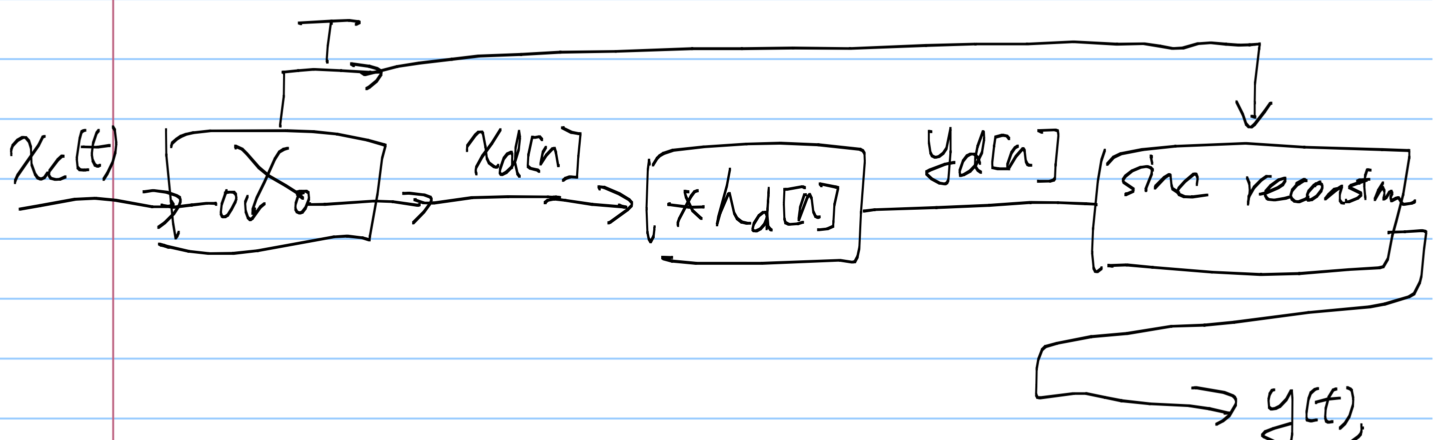
Therefore, if we can make sure $H_d(e^{j\omega})$ has the same spectrum as $H_{step2}(e^{j\omega})$, then the $Y_d(e^{j\omega})$ will be a stretched vesion of $Y_p(e^{j\omega})$.

We thus maintain the mirror relationship betwe, the conceptual & the practical systems.

$\Rightarrow$ The output of the practical system also fits the desired freq response $H(j\omega)$.

Step 3: Perform inverse DTFT on $H_{step2}(e^{j\omega})$ to generate $h_d[n]$

The end result becomes



which has the desired end to end $H(j\omega)/h(t)$.