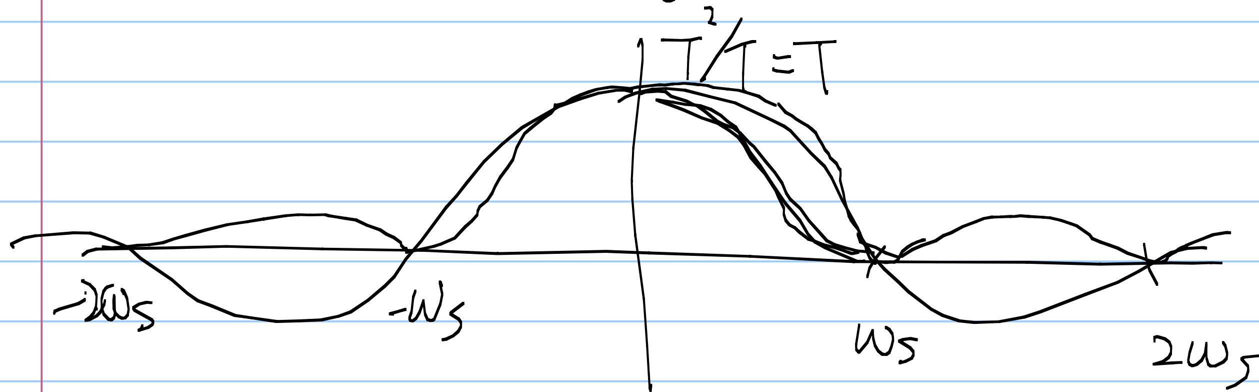
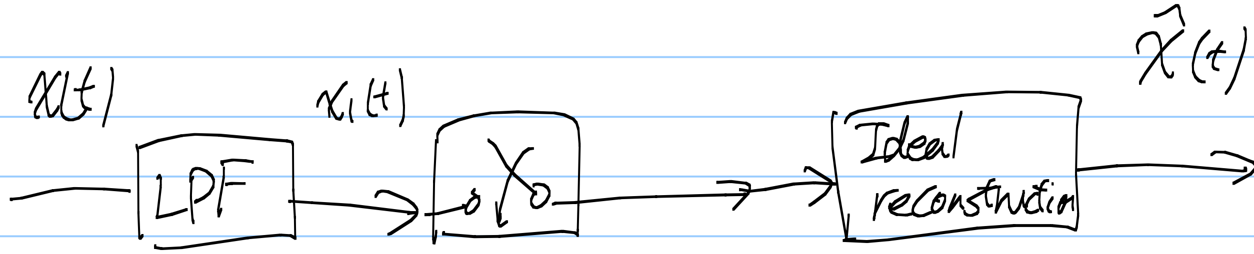
$$H_1(j\omega) = \frac{1}{T}\left(\frac{2\sin(\omega T/2)}{\omega}\right)^2$$

$$= \frac{1}{T}\left(H_0(j\omega)\right)^2$$



The $(\ )^2$ further suppresses the side copies (the freq with zero gain), which thus gives better approximation of the original signal.

We now know that when $\omega_s > 2\omega_m$, reconstruction can be perfect.



This is not possible.

$x_1(t) = \hat{x}(t)$.

However, in practice, we can only directly sample $x(t)$. What will happen when the original bandwidth is too large $\omega_m > \frac{\omega_s}{2}$?

(sampling is too slow)

or when $\omega_s < 2W_M$. In this case, our reconstruction will not lead to

$\hat{x}(t) = x(t)$. We say the system is <u>under-sampled</u>

Q: Can we predict $\hat{x}(t)$ even when the system is under-sampled?

Ans: Yes, the undersampling effect is termed <u>alliasing</u>

A special example:

$$x(t) = \cos(\omega_0 t + \phi)$$

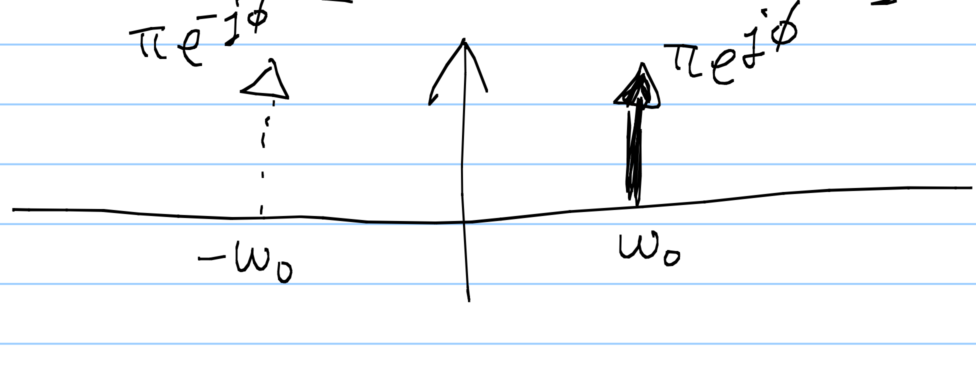
QI: Find FT of $x(t)$.

Ans: By noting that

$$x(t) = \frac{1}{2} e^{j(\omega_0 t + \phi)} + \frac{1}{2} e^{-j(\omega_0 t + \phi)}$$

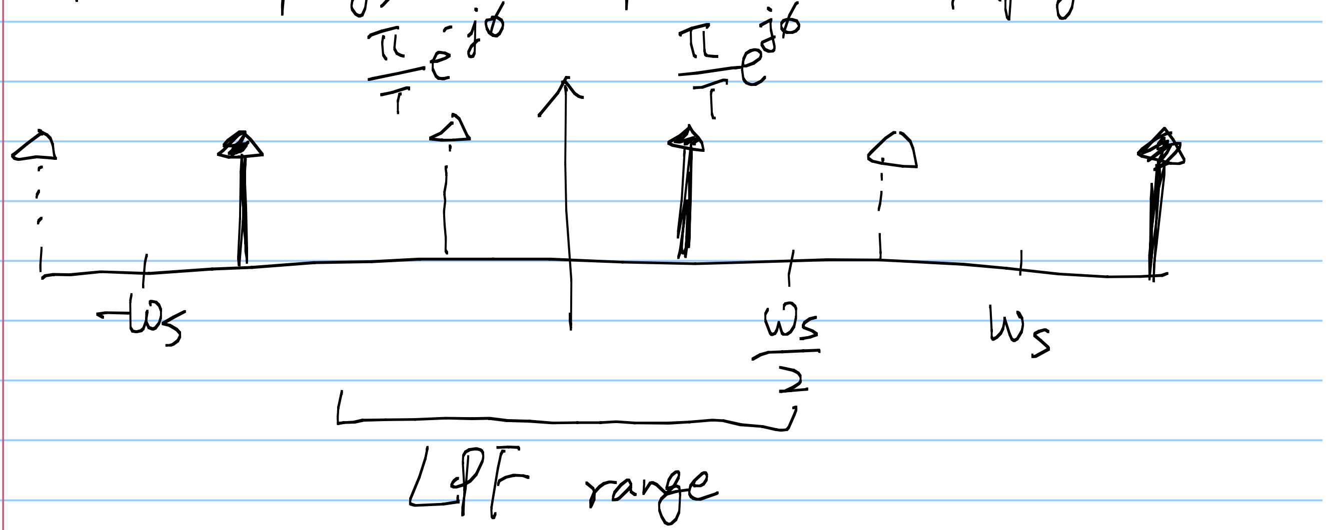
$$= \frac{1}{2} e^{j\phi} e^{j\omega_0 t} + \frac{1}{2} e^{-j\phi} e^{-j\omega_0 t}$$

$$X(j\omega) = \frac{2\pi}{2} e^{j\phi} \delta(\omega - \omega_0) + \frac{2\pi}{2} e^{-j\phi} \delta(\omega + \omega_0)$$



Sampling with $\omega_s > 2\omega_0$

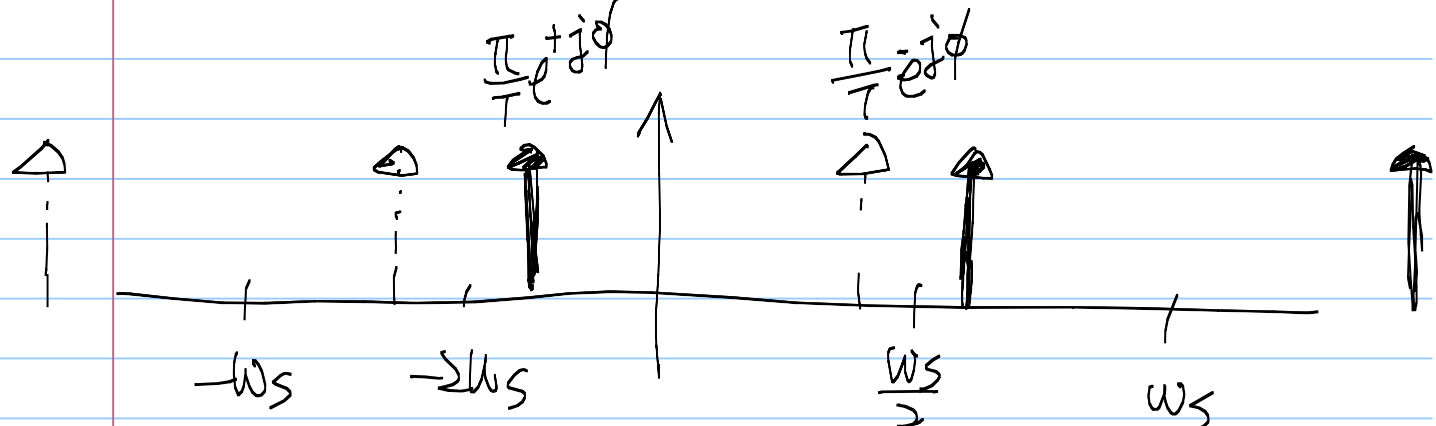
After sampling, the spectrum of $X_p(j\omega)$ becomes



The LPF will isolate the freq in this range.

However, if we keep increasing ω_0 while let ω_s be constant, eventually the system will be <u>under-sampled</u>.

We will have when $\omega_s < 2\omega_0$



Therefore, after the LPF and $\times T$, we have

$$\hat{X}(j\omega) = \pi e^{-j\phi} \delta(\omega - (\omega_s - \omega_0))$$
$$+ \pi e^{j\phi} \delta(\omega + (\omega_s - \omega_0))$$

$$\Rightarrow \hat{x}(t) = \frac{1}{2} e^{-j\phi} e^{j(\omega_s - \omega_0)t}$$
$$+ \frac{1}{2} e^{+j\phi} e^{j(-(\omega_s - \omega_0))t}$$
$$= \cos((\omega_s - \omega_0)t - \phi)$$

Summary

	$\omega_s > 2\omega_0$	$\omega_s < 2\omega_0$
$x(t) = \cos(\omega_0 t + \phi)$	$\hat{x}(t) = \cos(\omega_0 t + \phi)$	$\hat{x}(t) = \cos((\omega_s - \omega_0)t - \phi)$

Therefore, as ω_0 increases (speeds up), the freq of $\hat{x}(t)$ speeds up first but once $\omega_0 > \frac{\omega_s}{2}$ $\hat{x}(t)$ slows down $(\omega_s - \omega_0)$ & we have phase reversal $\phi \to (-\phi)$

This explains the stroboscope effect: Why in movies, wheels seem to turn slowly and backward.