* How come we can do perfect band-limited reconstruction

Ans: When the movement of the steering wheel is limited, there is only one route to drive through all the sampling points.

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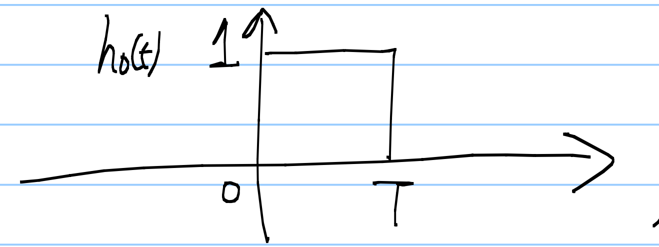
* The conceptual ITS $x_p(t)$ helps us devise the ideal band-limited interpolation.

And it also helps us to analyze the non-ideal reconstruction

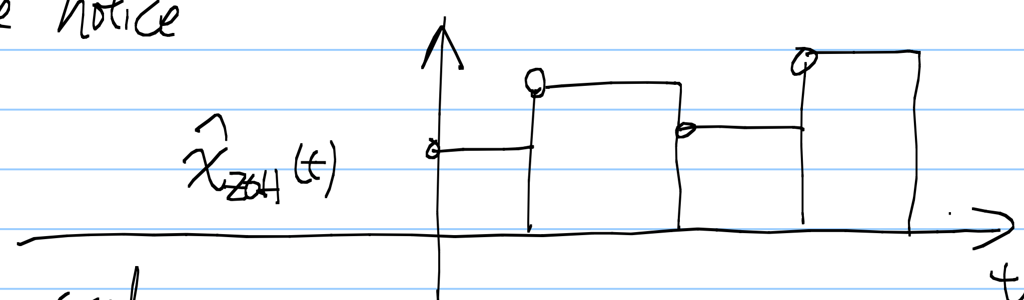
Ex: The Zero-Order Hold

If we define



,

then we notice



can be expressed as

$$\hat{x}_{ZOH}(t) = \sum_{n=-\infty}^{\infty} x[n]\, h_0(t-nT)$$

Recall $x_p(t) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t-nT)$

$\Rightarrow$ We can also view ZOH as follows

$$\hat{x}_{ZOH}(t) = \sum_{n=-\infty}^{\infty} x[n] \left(\delta(t-nT) * h_0(t) \right)$$

$$= \left(\sum_{n=-\infty}^{\infty} x[n]\,\delta(t-nT) \right) * h_0(t)$$

p.192

$$= x_p(t) * h_0(t)$$

For comparison, the optimal reconstruction is

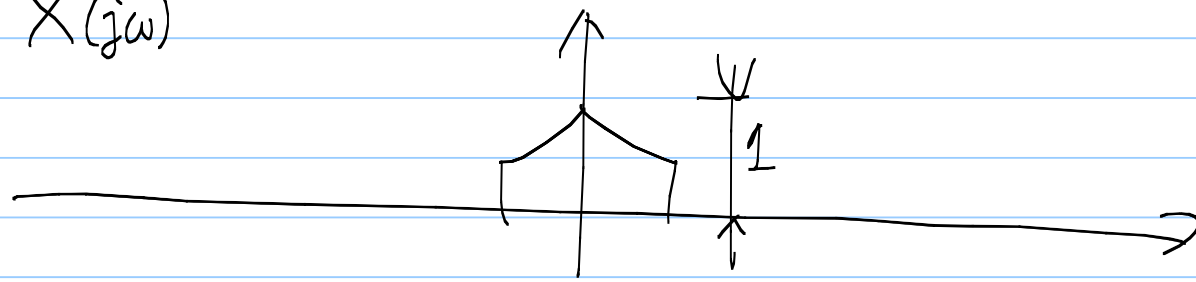
$$\hat{x}_{opt}(t) = x_p(t) * \left(T \cdot h_{LPF}(t)\right)$$

cut-off $\frac{\omega_s}{2}$

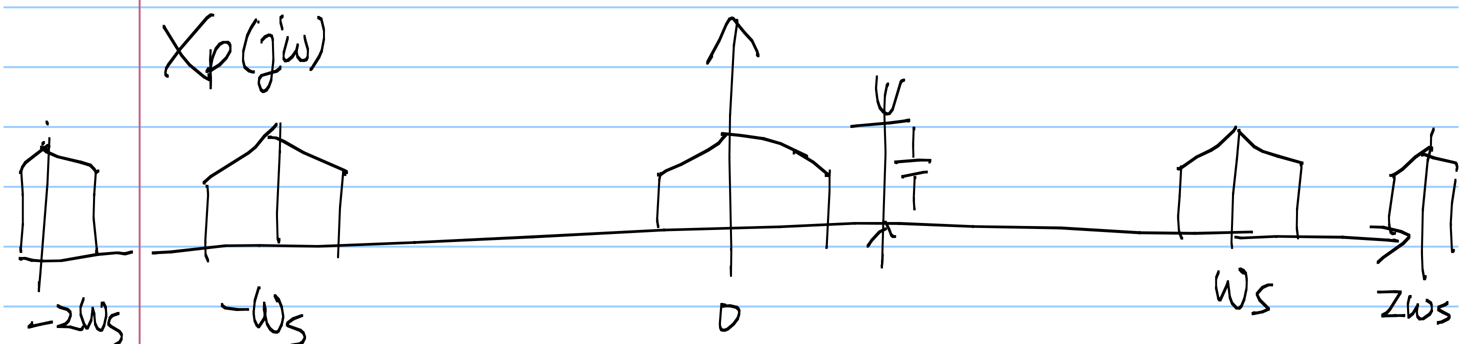
As a result,

$$\hat{X}_{ZOH}(j\omega) = X_p(j\omega) \cdot H_0(j\omega)$$

$X(j\omega)$



$X_p(j\omega)$

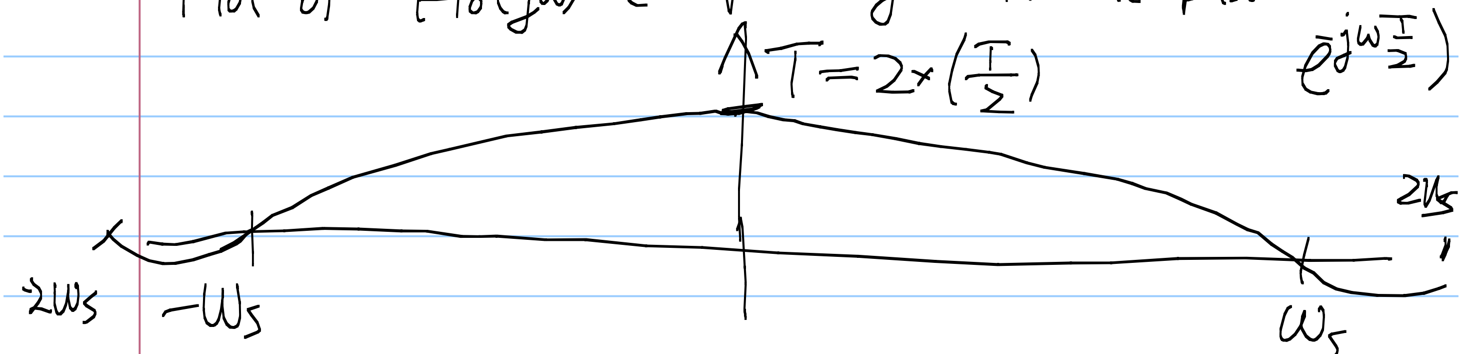


$$H_0(j\omega) = ?$$

$$= \underline{\underline{e^{-j\omega\frac{T}{2}}}} \underline{\underline{\left(\frac{2\sin(\omega\frac{T}{2})}{\omega}\right)}}$$

Time-shift Example 4.4

Plot of $H_0(j\omega)$ (temporarily omit the phase shift $e^{-j\omega\frac{T}{2}}$)

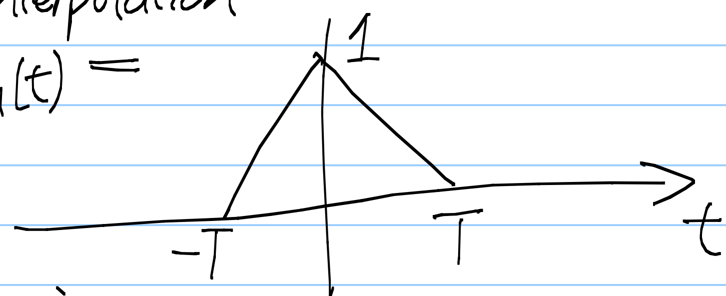


also
ZOH is ^ trying to keep the center copy of the freq spectrum while suppressing side copies.

P. 524 Fig. 7.11.
But not all side copies can be completely suppressed.

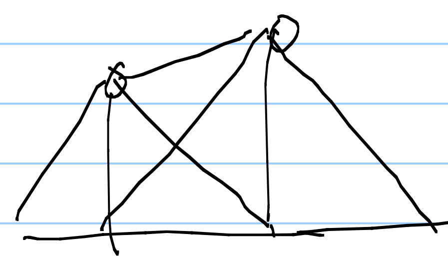
* Linear Interpolation

Define $h_1(t) =$



then we notice

Linear connection



therefore we have

$$\hat{x}_{LIN}(t) = \sum_{n=-\infty}^{\infty} x[n]\, h_1(t-nT) \quad \text{where}$$

By the same reason as the ZOH derivation

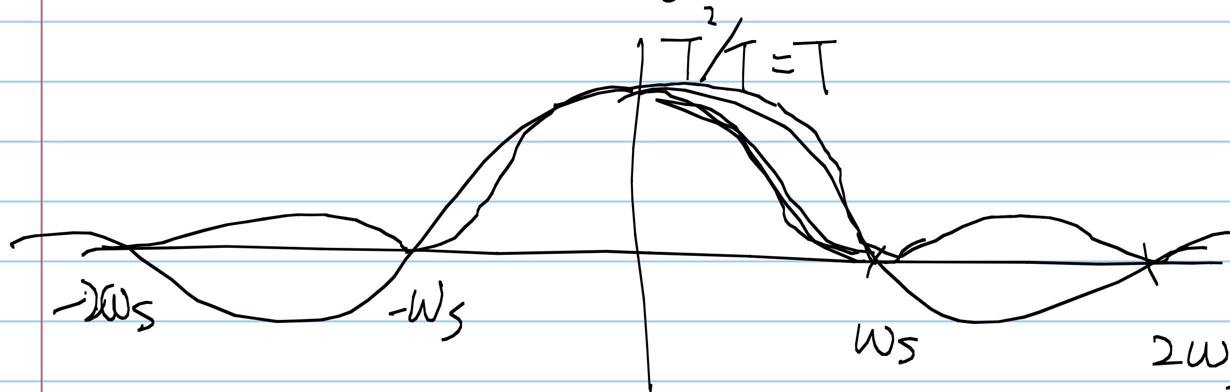
$$= x_p(t) * h_1(t)$$

$$\hat{X}_{LIN}(j\omega) = X_p(j\omega)\, H_1(j\omega)$$

Q: What is $H_1(j\omega)$? (Direct Computation)

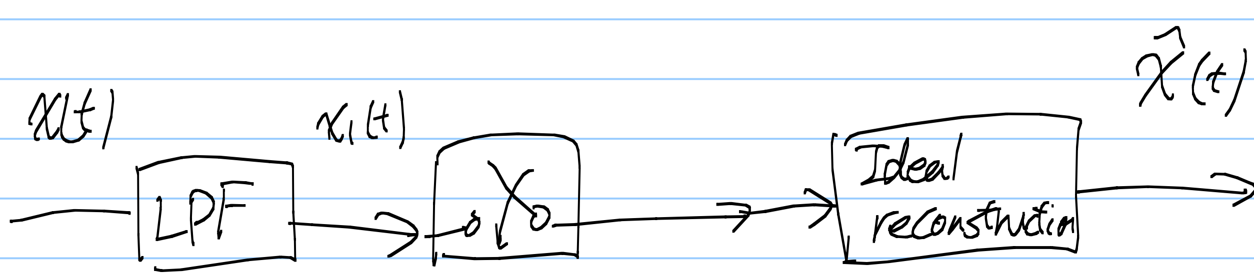
$$H_1(j\omega) = \frac{1}{T}\left(\frac{2\sin(\omega T/2)}{\omega}\right)^2$$

$$= \frac{1}{T}\left(H_0(j\omega)\right)^2$$



The $(\ )^2$ further suppresses the side copies (the freq with zero gain), which thus gives better approximation of the original signal.

We now know that when $\omega_s > 2\omega_M$, reconstruction can be perfect.



↑ This is not possible. $\qquad x_1(t) = \hat{x}(t)$.

However, in practice, we can only directly sample $x(t)$. What will happen when the original bandwidth is too large $\omega_M > \frac{\omega_s}{2}$?