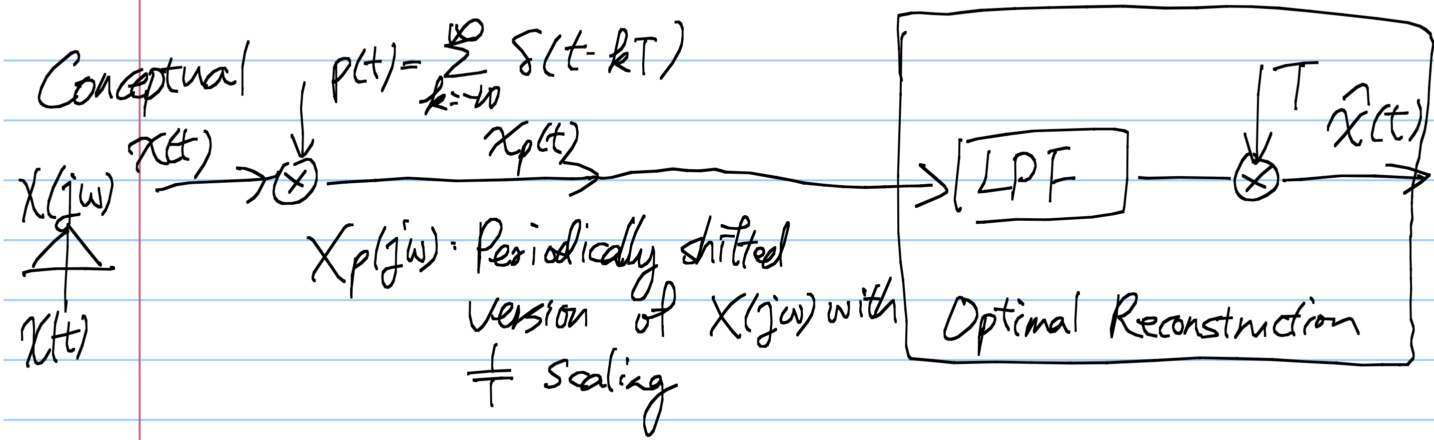
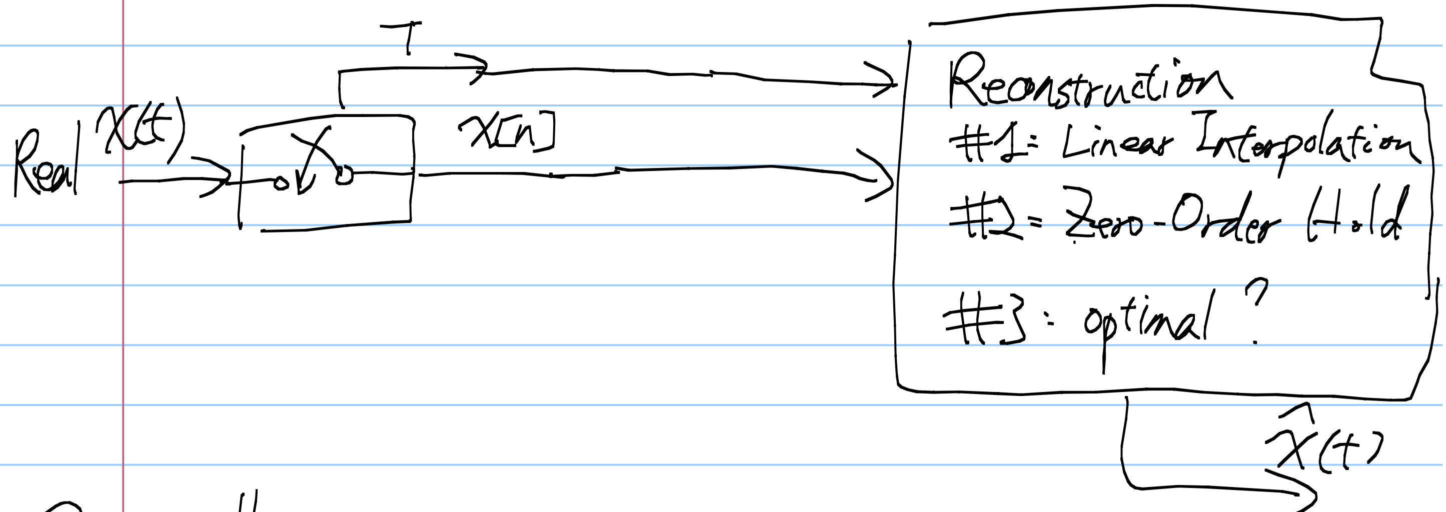
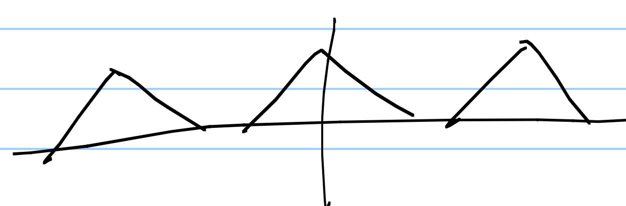




Conceptually,
The optimal reconstruction works if

① $x(t)$ is band-limited $X(j\omega) = 0$ if $|\omega| > W_M$

② $W_s > 2W_M$



(3.1) The cut-off freq of the LPF is

$$W_M < W_{cut\text{-}off} < W_s - W_M$$

In practice, we simply choose

or (3.2) $W_{cut\text{-}off} = \frac{1}{2} W_s$.

Q: How do we implement the conceptually optimal reconstruction in practice?

* Let's ~~take~~ a closer look at the LPF-based reconstruction

$$\hat{x}(t) = T \cdot x_p(t) * \frac{\sin(\frac{\omega_s}{2}t)}{\pi t}$$

$\frac{\omega_s}{2}$ is the cutoff freq of LPF

$$= T \cdot \left(\sum_{k=-\infty}^{\infty} x(nT)\,\delta(t-kT) \right) * \frac{\sin(\frac{\omega_s}{2}t)}{\pi t}$$

Convolution of a shifted delta

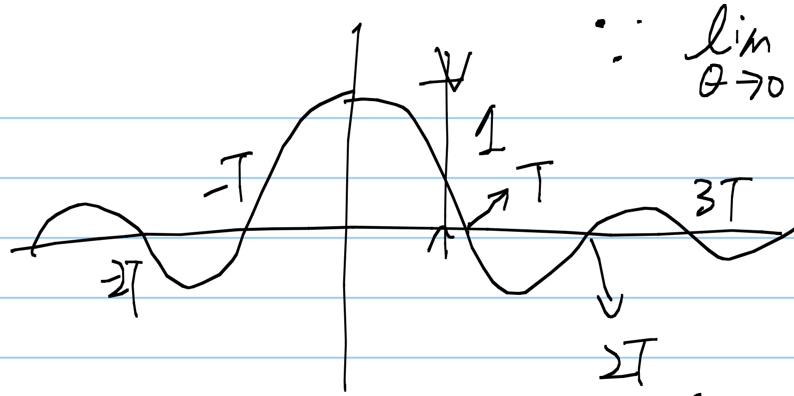
$\equiv$ Shift ~~of~~ the original signal

$$= \sum_{k=-\infty}^{\infty} x(nT) \left(T \cdot \frac{\sin(\frac{\omega_s}{2}(t-kT))}{\pi(t-kT)} \right)$$

Since $\omega_s = \frac{2\pi}{T}$

$$\Rightarrow = \sum_{k=-\infty}^{\infty} \underbrace{x(nT)}_{x[n]} \cdot \frac{\sin(\frac{\pi}{T}(t-kT))}{\frac{\pi}{T}(t-kT)}$$

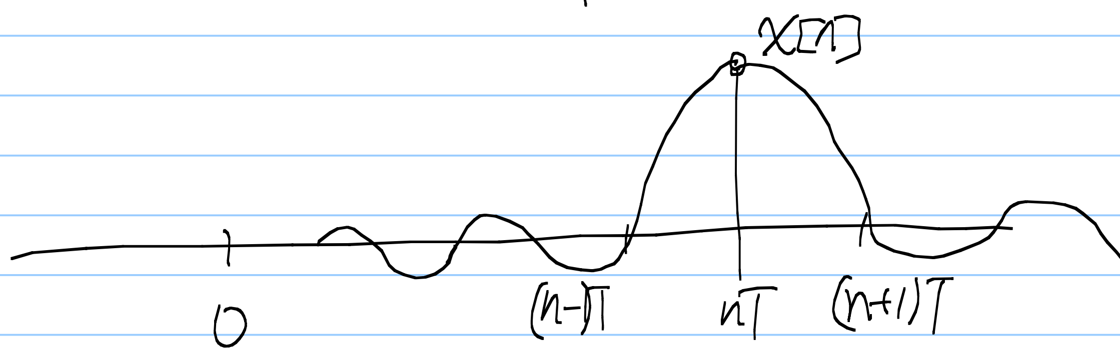
How to plot $\frac{\sin(\frac{\pi}{T}t)}{\frac{\pi}{T}t}$?



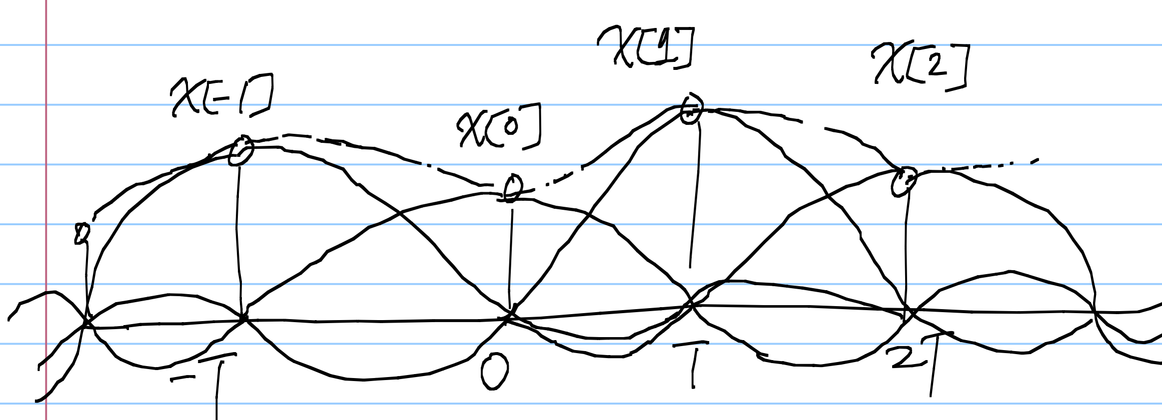
$\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\because \frac{\omega_s}{2} \cdot t = \pi, 2\pi, \cdots$

Thus $x[n] \frac{\sin\left(\frac{\pi}{T}(t-nT)\right)}{\frac{\pi}{T}(t-nT)}$ is



The final reconstruction (practical)



※ Optimal Reconstruction in practice:
Superposition of many shifted, properly scaled sinc functions. $\Longrightarrow$ termed <u>Band-limited Interpolation</u>.

* How come we can do perfect band-limited reconstruction

Ans: When the movement of the steering wheel is limited, there is only one route to drive through all the sampling points.

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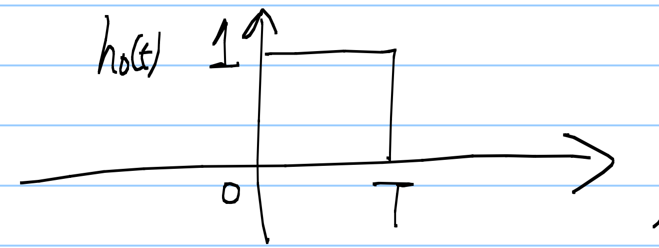
* The conceptual ITS $x_p(t)$ helps us devise the ideal band-limited interpolation.

And it also helps us to analyze the non-ideal reconstruction

Ex: The Zero-Order Hold

If we define $h_0(t)$



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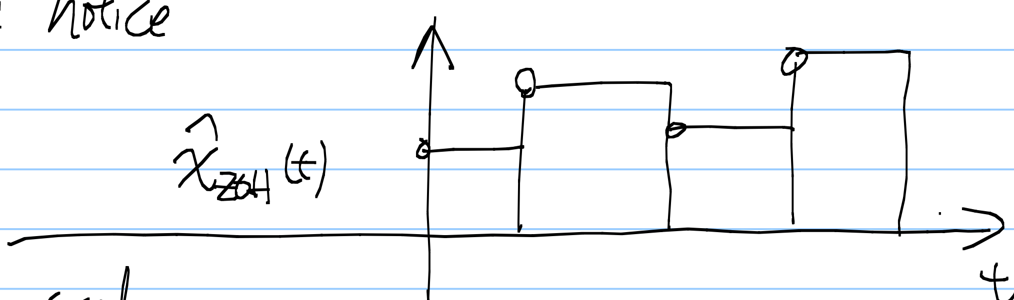
then we notice



can be expressed as

$$\hat{x}_{ZOH}(t) = \sum_{n=-\infty}^{\infty} x[n]\, h_0(t-nT)$$

Recall $x_p(t) = \sum_{n=-\infty}^{\infty} x[n]\, \delta(t-nT)$

$\Rightarrow$ We can also view ZOH as follows

$$\hat{x}_{ZOH}(t) = \sum_{n=-\infty}^{\infty} x[n] \left(\delta(t-nT) * h_0(t) \right)$$

$$= \left(\sum_{n=-\infty}^{\infty} x[n]\, \delta(t-nT) \right) * h_0(t)$$