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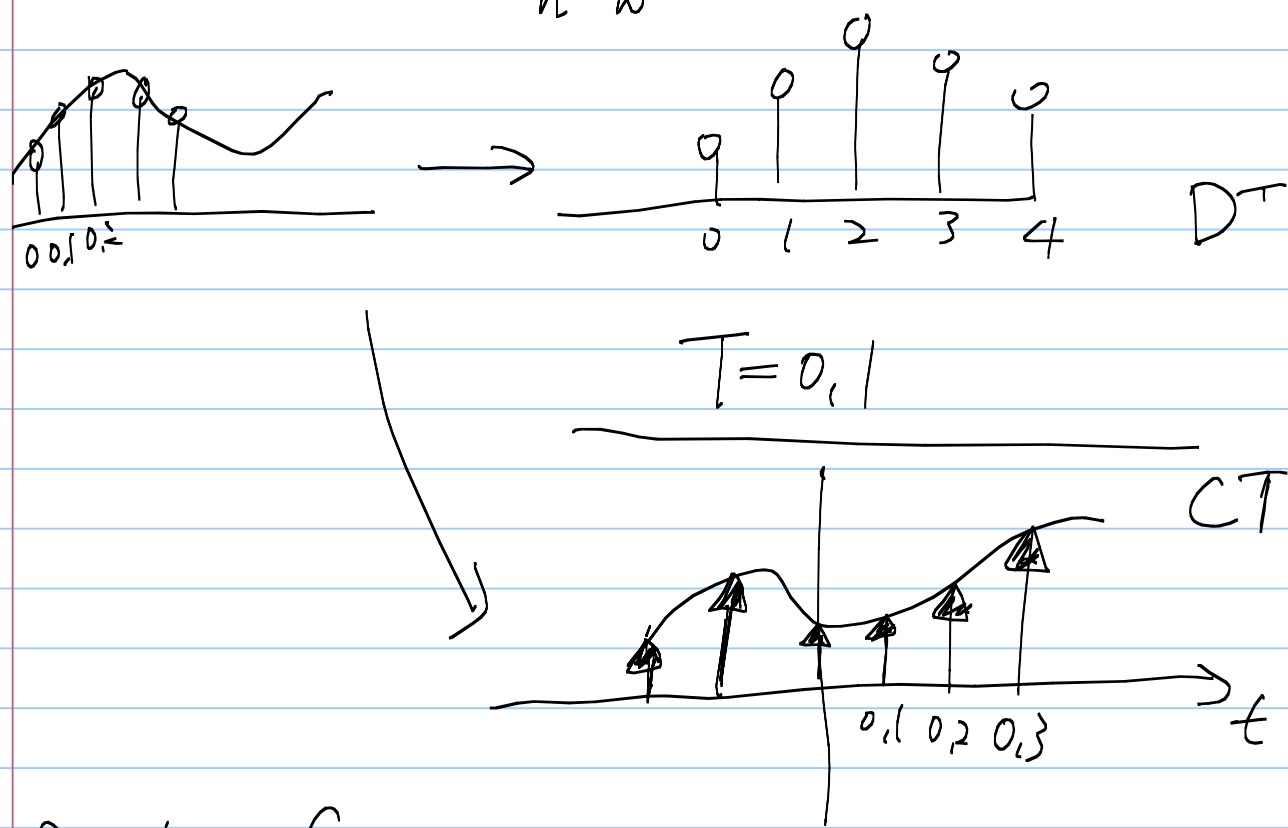
More explicitly, a band-limited $x(t)$ (through a LPF) can be perfectly reconstructed.

To formalize this idea, we consider a new type of sampling: Impulse-train sampling (ITS) (vs. traditional sampling)

$$x(t) \to \begin{cases} x[n] \\ T \end{cases}$$

$$x(t) \to x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$



Remark 1: $\begin{cases} x[n] \\ T \end{cases}$ and $x_p(t)$ present the same amount of info. ⇒ They are equivalent.

Remark 2: Traditional sampling ⟹ DT
ITS ⟹ CT

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Remark 3: ITS is a conceptual tool, not implementable in practice.

But as we will see, $x_p(t)$ (ITS) is more convenient for analysis

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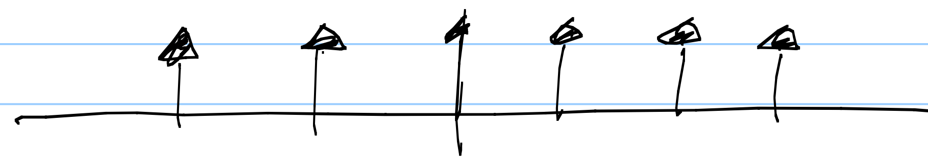
* Let us focus more on ITS. Conceptually, how do we perform ITS?

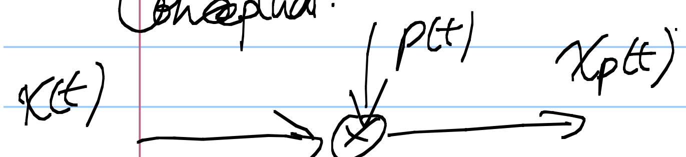
Let $p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$ be an impulse train



Then
$$x(t)\cdot p(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT)$$
$$= \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT)$$
$$= \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT) = x_p(t)$$

$\Rightarrow$ ITS is the product of $x(t)$, $p(t)$ in time.

Conceptual:

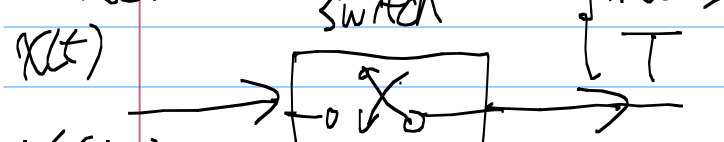


$X(j\omega)$

$X_p(j\omega) = ?$

$\boxed{\text{Let us focus on the freq domain of the systems}}$

Real



$X(j\omega)$

Q: What is the CTFT of $x_p(t)$?

Ans: We know $X_p(j\omega) = \frac{1}{2\pi}\left[X(j\omega) * P(j\omega)\right]$

By table-lookup (or by CTFS + inspection

$$P(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - k\frac{2\pi}{T})$$

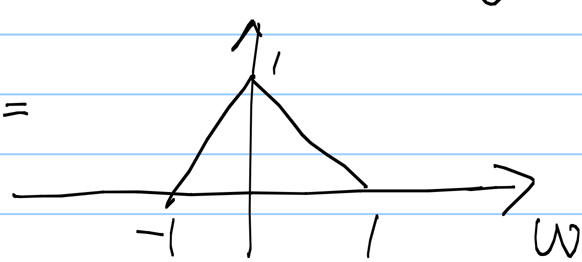
$$\Rightarrow X_p(j\omega) = \frac{1}{2\pi}\frac{2\pi}{T}\left[X(j\omega) * \sum_{k=-\infty}^{\infty}\delta(\omega - k\underbrace{\frac{2\pi}{T}}_{\omega_s})\right]$$

$$\boxed{\begin{array}{l}\text{Convolution of a shifted delta} \\ \equiv \text{Shift the original signal}\end{array}}$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

In sum, the freq spectrum $X_p(j\omega)$ of $x_p(t)$ is the periodically shifted version of $X(j\omega)$ with period $\omega_s = \frac{2\pi}{T}$ & a scaling factor $\frac{1}{T}$.

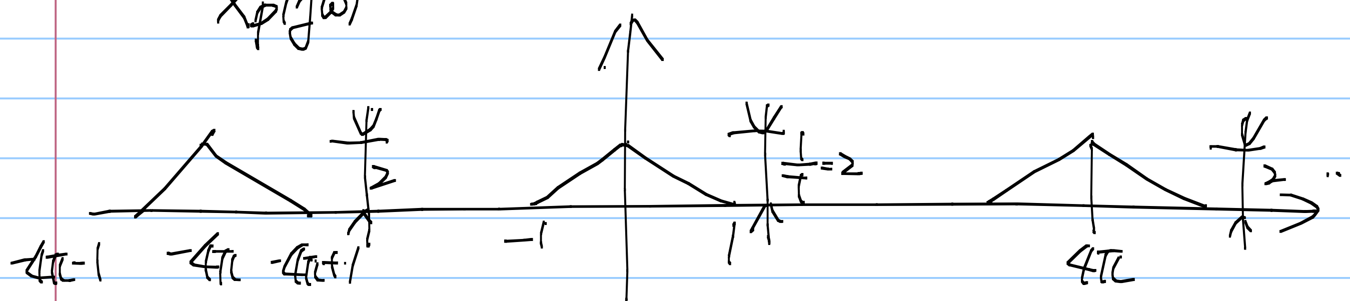
Example: $X(j\omega) =$



Consider a sampling freq 2Hz, what

is the $X_p(j\omega)$, the CTFT of its ITS $x_p(t)$.

Ans: $T = \frac{1}{2}$, $\omega_s = \frac{2\pi}{1/2} = 4\pi$



Q: Conceptually, how to recover $x(t)$ from $x_p(t)$?

Ans: Step 1: Pass it through a LPF with cut-off freq $= 1$.

Step 2: Multiply T.

* Side note: Compare it with AM

AM

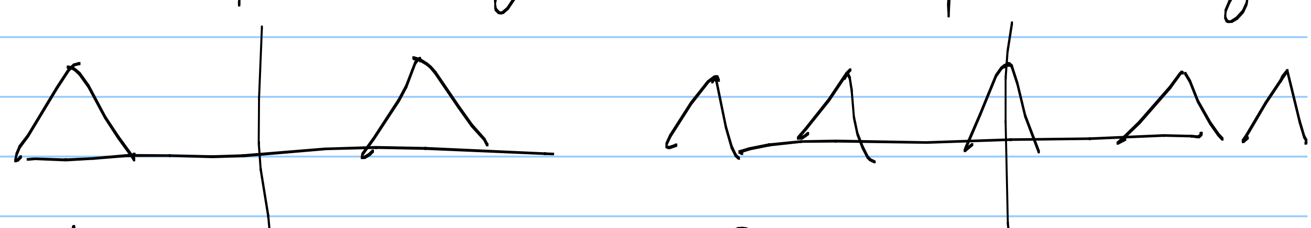
$y(t) = x(t) \times \cos(\omega_c t)$

Two copies in freq.

ITS

$x_p(t) = x(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t - kT)$

∞ copies in freq



Demod: ① $\times \cos(\omega_c t)$

② LPF

③ $\times 2$

Reconstruction ① LPF

② $\times T$

(cont'd)
To ensure perfect reconstruction, we need

AM	ITS
$\omega_c > \underline{W_M}$	$\omega_s > 2W_M$
	$\therefore W_M < \omega_s - W_M$

The band-limited signal

Otherwise we will have freq overlap.

☆ Sampling theorem

Let $x(t)$ be a band-limited signal s.t. $|X(j\omega)| = 0$ if $|\omega| > W_M$

Then $x(t)$ can be perfectly reconstructed from its samples provided the sampling freq ω_s satisfies

$$\omega_s > \underbrace{2W_M}_{}$$

Also termed the Nyquist (sampling) freq.

If ω_s is large (sampling fast enough when compared to the underlying signal.) we can reconstruct the original signal.