Section 7 Sampling (discrete-time signal processing)

Q Given a CT impulse response $h(t)$, how to implement it.

Method 1: A conti—time approach by capacitors, resistance, etc.

Hard. Most of the time we can only approximat it.

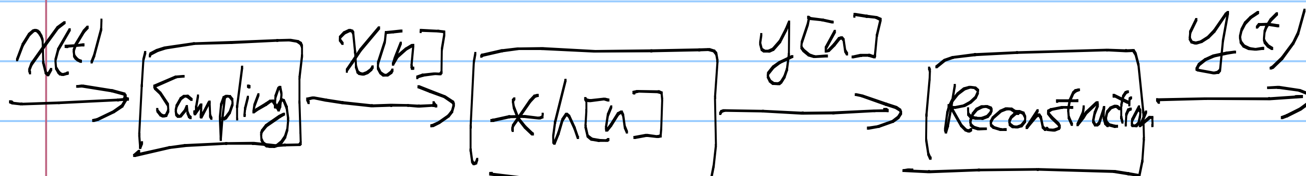
Method 2: ① Sampling a CT $x(t)$ to $x[n] = x(nT)$

② Do discrete-time processing

$$y[n] = h[n] * x[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

(Involving only multiplication & addition)

③ Reconstruct a CT signal $y(t)$ from $y[n]$



Relatively easy

New components.

Hopefully the overall system has the desired $\boxed{h(t)}$

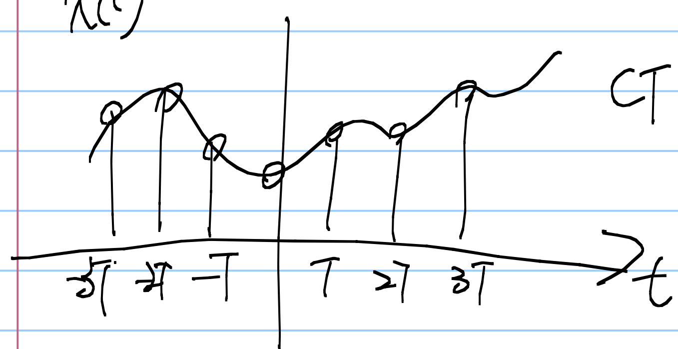
Before we continue, let's focus on the basic reconstruction system. (No processing between sampling & reconstruction.)

$$x(t) \rightarrow \boxed{\text{Sampling}} \xrightarrow{x[n]} \boxed{\text{Reconstruction}} \rightarrow \hat{x}(t)$$

Q: How to perform sampling & reconstruction in the best way?

Q: What is sampling?

$x(t)$



$$\text{CT } x(t) \xrightarrow{\text{Sampling}} \begin{cases} x[n] = x(nT) & \text{value array} \\ T & \text{spacing} \end{cases}$$

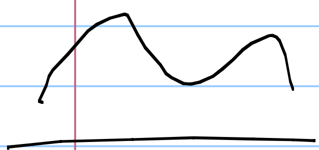
* A continuous curve $x(t)$ is now converted to a <u>value array</u> & a single <u>spacing value T</u>.

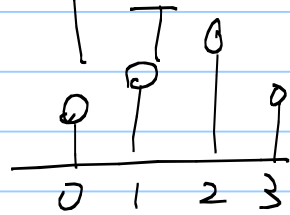
$x[n]$ is the n-th sample value

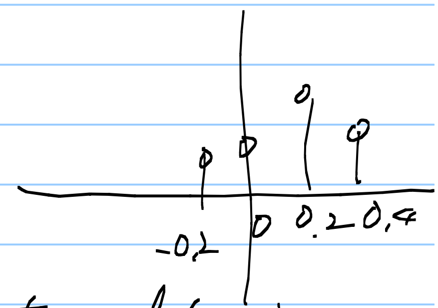
T is the sampling period

$2\pi/T$ is the sampling freq

$$x(t) \xrightarrow{\text{sampling}} x[n] = x(nT) \xrightarrow{\text{reconstruction}} \hat{x}(t)$$







$T = 0.2$ sec

The goal of reconstruction is to determine $\hat{x}(t)$ when $t \neq nT$ ex: $\hat{x}(\pi)$

* How to connect the dots?

Method 1: Linear Interpolation: Connecting the dots by line segments.

Advantage: ① Easy to implement

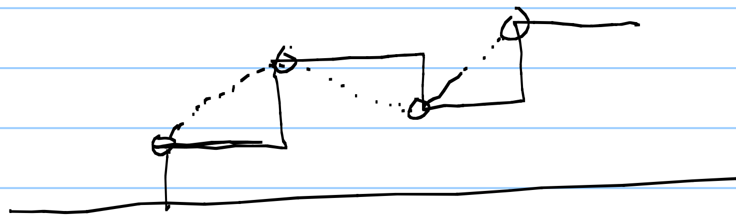
② When T is small or $\omega_s = \frac{2\pi}{T}$ is large.

it approximates $x(t)$ very well.

Disadvantage: When the sampling freq is small $\frac{2\pi}{T}$ is small, we lose too much detail.

Method 2: zero-order hold.

Hold the sample value until the next sample point.



Advantage: ① The easiest

② When $\frac{2\pi}{T}$ is large, it's good.

Disadvantage: The same as linear interpolation

Question: Is there always loss of information during sampling?

If not, under what condition can $x(t)$ be perfectly reconstructed from $\{x[n]\}_T$ & how?

Observation: What is causing the problem?

$x(t)$ oscillates too fast s.t. the fast movement cannot be captured by the equally spaced samples.

Intuitively,

A slowly moving $x(t)$ can be reconstructed

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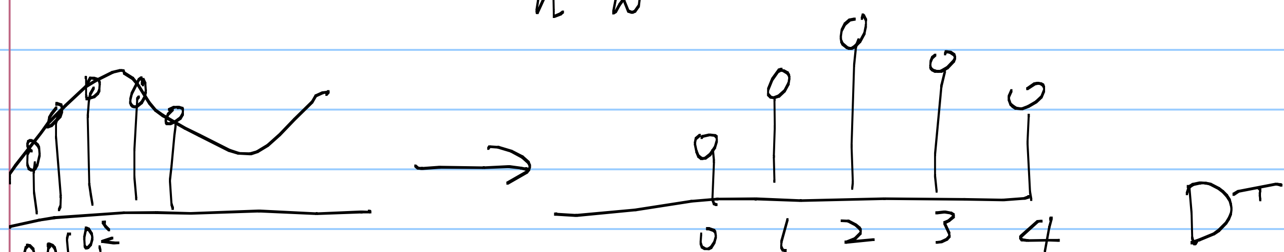
More explicitly, a band-limited $x(t)$ (through a LPF) can be perfectly reconstructed.

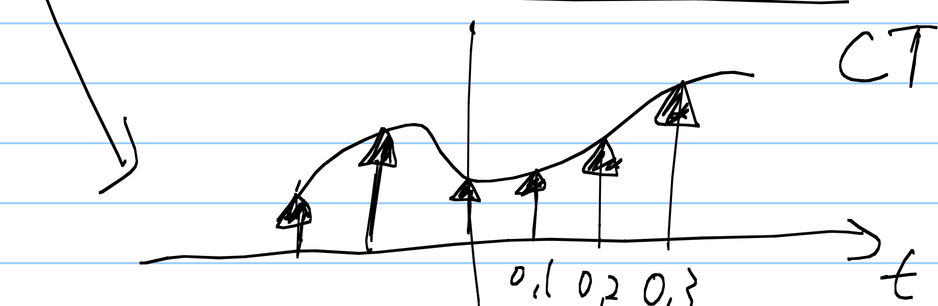
To formalize this idea, we consider a new type of sampling: Impulse-train sampling (ITS) (vs. traditional sampling)

$$x(t) \rightarrow \begin{cases} x[n] \\ T \end{cases}$$

$$x(t) \rightarrow x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$



$T = 0.1$



Remark 1: $\begin{cases} x[n] \\ T \end{cases}$ and $x_p(t)$ present the same amount of info. ⇒ They are equivalent.

Remark 2: Traditional sampling ⇒ DT
ITS ⇒ CT