* A final example: Solving difference equation

$$6y[n]-5y[n-1]+y[n-2]=18x[n]-8x[n-1]$$

Find $h[n]$

Ans: Take DTFT.

$$6Y(e^{j\omega n})-5e^{-j\omega\cdot 1}Y(e^{j\omega})+e^{-j\omega\cdot 2}Y(e^{j\omega})$$
$$=18X(e^{j\omega})-8e^{-j\omega}X(e^{j\omega})$$

We know $Y(e^{j\omega})=X(e^{j\omega})\cdot H(e^{j\omega})$

$$\Rightarrow H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{18-8e^{-j\omega}}{6-5e^{-j\omega}+e^{-j\omega\cdot 2}}$$

By partial fraction

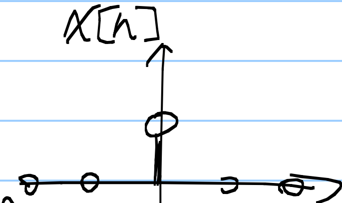
$$H(e^{j\omega})=\frac{2}{1-\frac{1}{3}e^{-j\omega}}+\frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

By table look up p.391 Table 5.1

$$\Rightarrow h[n]=2\left(\frac{1}{3}\right)^{+n}u[n]+\left(\frac{1}{2}\right)^{n}u[n]$$ #

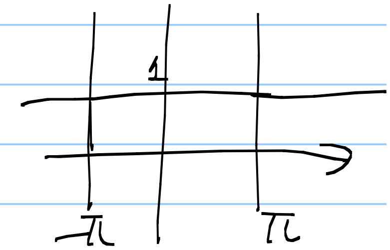
* Important pairs of DTFT.

① $x[n] = \delta[n]$



Direct computation

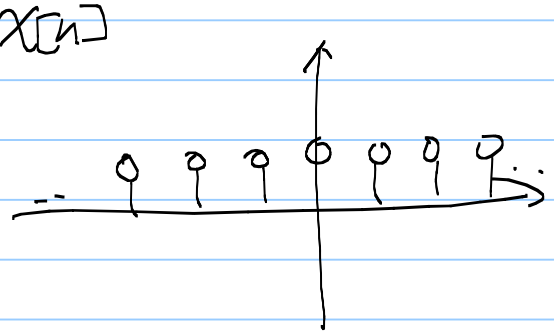
$X(e^{j\omega}) = \begin{cases} 1 & \text{if } -\pi < \omega < \pi \\ \text{periodic} & \end{cases}$

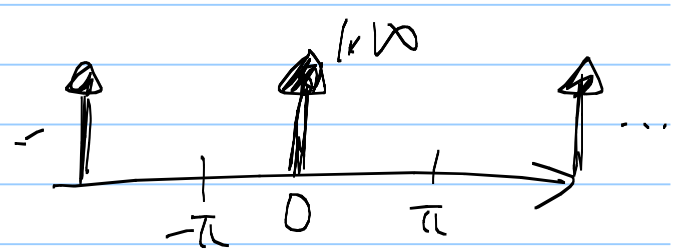


② $X(e^{j\omega}) = \begin{cases} \delta(\omega) & \text{if } -\pi < \omega < \pi \\ \text{periodic with period } 2\pi. & \end{cases}$

direct computation

$x[n] = \frac{1}{2\pi}$





③ $x[n] = e^{j\omega_c n}$

$X(e^{j\omega}) = \begin{cases} 2\pi\,\delta(\omega - \underline{\omega_c - 2\pi k}) & \text{if } -\pi < \omega < \pi \\ \text{periodic with period } 2\pi. & \end{cases}$

move ω_c to the right range

Inspection or by freq-shift property

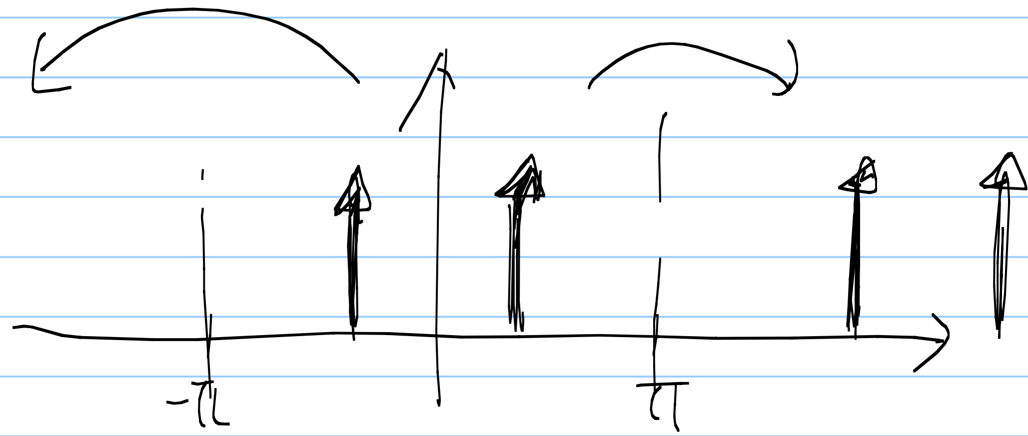
p.176

(3.1) $x[n] = \cos(\omega_c n)$

choose k to make it within $[-\pi, \pi)$

Inspection & periodicity

$$X(e^{j\omega}) = \begin{cases} \pi\delta(\omega - (\omega_c + 2k\pi)) + \pi\delta(\omega + (\omega_c + 2k\pi)) & \text{if } -\pi < \omega < \pi \\ \text{periodic with period } 2\pi \end{cases}$$



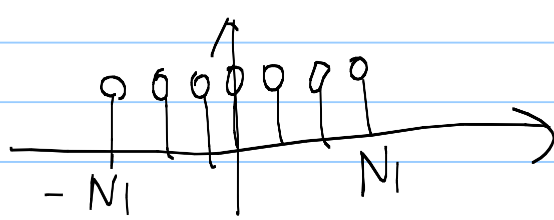
No easy formula.

Exercise

$x[n] = \sin(\omega_c \cdot n)$

(4) $x[n] = u[n + N_1] - u[n - (N_1 + 1)]$

direct computation



$$X(e^{j\omega}) = \frac{\sin(\omega(N_1 + \frac{1}{2}))}{\sin(\omega/2)}$$

⑤ $X(e^{j\omega}) = \begin{cases} \text{periodic w. period } 2\pi \\ U(\omega+W) - U(\omega-W) & \text{if } |\omega|<\pi \end{cases}$

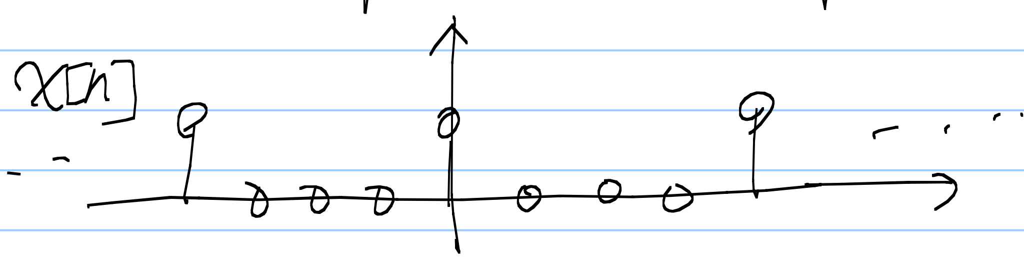
direct computation

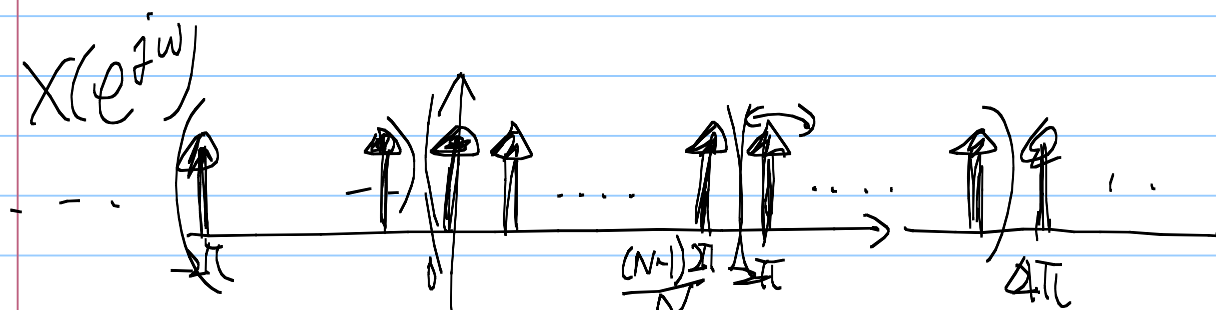
$$x[n] = \frac{\sin(Wn)}{\pi n}$$

⑥ $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-kN]$

generalized DTFS

$$X(e^{j\omega}) = \begin{cases} \sum_{k=1}^{N-1} \frac{2\pi}{N}\,\delta\left(\omega - \frac{2\pi k}{N}\right) & \text{if } 0\le\omega<2\pi \\ \text{periodic with period } 2\pi \end{cases}$$





Be careful about the distinction between

$\delta[n]$ & $\delta(\omega)$

* Duality: See Sec 5.7

Note Title 4/7/2010

An important summary ☆☆ (Fast Fourie Transform)

CTFS

time: conti $\leftrightarrow$ freq: aperiodic

time: periodic $\leftrightarrow$ freq: discrete

DTFS

time: discrete $\leftrightarrow$ freq: periodic

time: periodic $\leftrightarrow$ freq: discrete

☆☆ CTFT

time: conti $\leftrightarrow$ freq: aperiodic

time: aperiodic $\leftrightarrow$ freq: conti

DTFT

time: discrete $\leftrightarrow$ freq: periodic

time: aperiodic $\leftrightarrow$ freq: conti

Time		Freq
Conti / discrete	$\longleftrightarrow$	aperiodic / period
aperiodic / periodic	$\longleftrightarrow$	conti / discrete

CTFS:
$$\begin{cases} a_k = \frac{1}{T}\int_T x(t) e^{-jk\frac{2\pi}{T}t} dt \\ x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t} \end{cases}$$

DTFS:
$$\begin{cases} a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk\frac{2\pi}{N}n} \\ x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n} \end{cases}$$

CTFT:
$$\begin{cases} X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt \\ x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw) e^{+jwt} dw \end{cases}$$

DTFT:
$$\begin{cases} X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn} \\ x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw}) e^{jwn} dw \end{cases}$$