We thus say this method of modulation

$y(t) = x(t)\cos(\omega_0 t)$ is Amplitude Modulation

In an AM radio, ω_0 ranges from

500 kHz to 1.6 MHz.

$\boxed{\text{AM}}$

We will talk more about AM in Chapter 8.

* 2nd Application of the multiplication property

Computing FT using the product form

Example $x(t) = \dfrac{\sin t \sin t/2}{\pi t^2}$

Find $X(j\omega)$

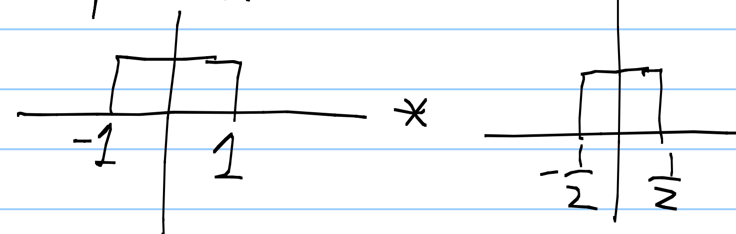
Ans: We notice that

$$x(t) = \pi\left(\frac{\sin t}{\pi t}\right)\left(\frac{\sin(t/2)}{\pi t}\right)$$

$$\Rightarrow X(j\omega) = \frac{1}{2\pi}\times\pi\ \mathcal{F}\left(\frac{\sin t}{\pi t}\right) * \mathcal{F}\left(\frac{\sin(t/2)}{\pi t}\right)$$

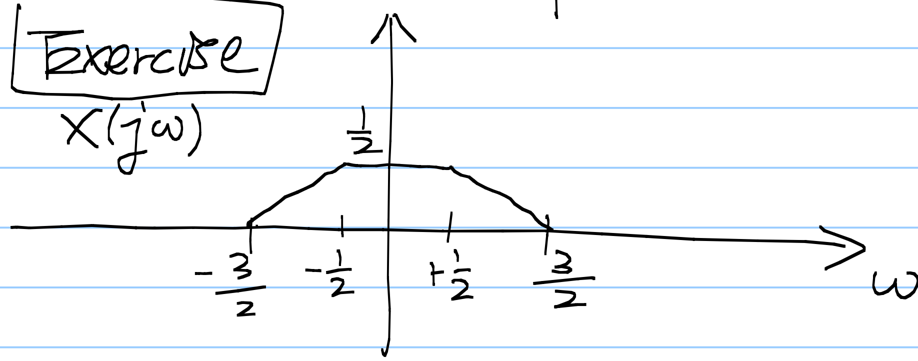
Again by Example 4.5

$X(j\omega) = \frac{1}{2}$



$\boxed{\text{Exercise}}$

$X(j\omega)$



* Differential Equations & F.T.

Q: consider a differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$$

Knowing $x(t)$, how to find $y(t)$

☆ How to use an LTI sys to solve this ordinary differential equation?

Sol'n: Step 1: Compute $H(j\omega)$

Take FT on both sides

$$\sum_{k=0}^{N} a_k (j\omega)^k Y(j\omega) = \sum_{k=0}^{M} b_k (j\omega)^k X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k}$$

* Once we find $H(j\omega)$, we can compute output $y(t)$ for any given input $x(t)$ by

$$\begin{tikzcd} x(t) \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & y(t) \arrow[d, leftrightarrow] \\ X(j\omega) \arrow[r, leftrightarrow, "\cdot H(j\omega)"'] & Y(j\omega) \end{tikzcd}$$

Example $\dfrac{dy(t)}{dt} + a\,y(t) = x(t)$ where $a>0$

find $h(t)$, $H(j\omega)$?

Ans: Take F.T.

$$j\omega Y(j\omega) + aY(j\omega) = X(j\omega)$$

$$H(j\omega) = \frac{1}{a+j\omega}$$

How to find $h(t)$? $h(t) = e^{-at}u(t)$

Ans: ① Too hard for direct computation in this example.

② Can be solved by table look-up (Table 4.2)

Review

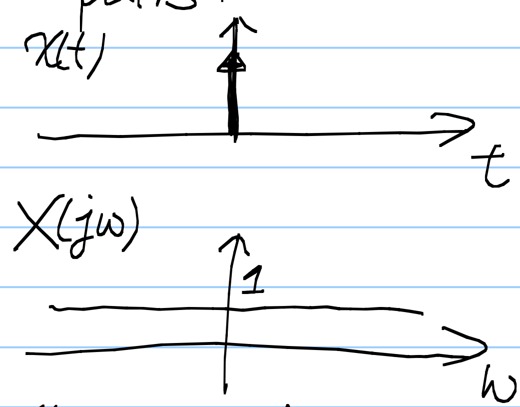
* CT FT:

A signal can now be represented in either time or freq domain.

* 6 important F.T pairs.

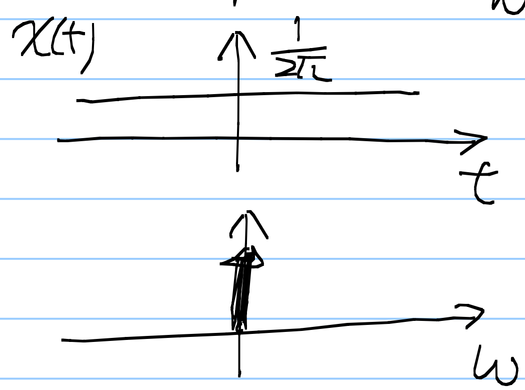
① $x(t) = \delta(t)$ — delta/impulse

$X(j\omega) = 1$ — const



② $x(t) = \frac{1}{2\pi}$ — const

$X(j\omega) = \delta(\omega)$ — delta/impulse



③ Example 4.4 $\boxed{\text{Sup}}$

$x(t) = u(t+T_1) - u(t-t_1)$ — rectangle

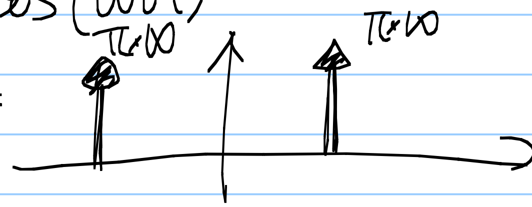
$X(j\omega) = \frac{2\sin \omega T_1}{\omega}$ — sinc

④ Example 4.5 $\boxed{\text{Sup}}$

$x(t) = \frac{\sin Wt}{\pi t}$ — sinc

$X(j\omega) = u(\omega + W) - u(\omega - W)$ — rectangle

⑤ $x(t) = \cos(\omega_0 t)$

$X(j\omega) =$ 

$= \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$

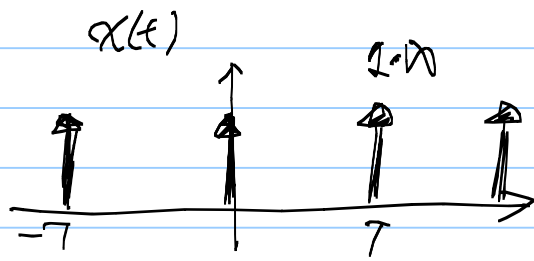
$x(t) = \sin(\omega_0 t)$

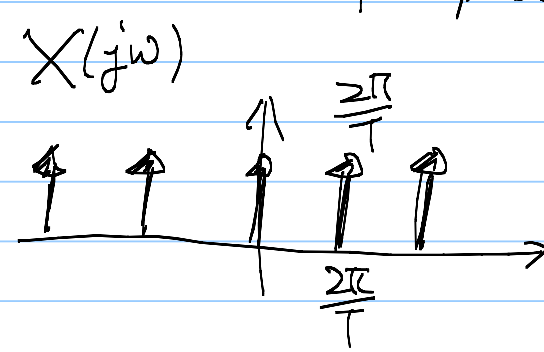
$$X(j\omega) = \frac{\pi}{j}\,\delta(\omega-\omega_0) - \frac{\pi}{j}\,\delta(\omega+\omega_0)$$

⑥ Example 4.8 $\boxed{\text{sup}}$

$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$ —— a train of impulses

$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)$ —— a train of impulses





(6.1) $x(t) = \frac{1}{W} \sum_{k=-\infty}^{\infty} \delta\left(t - k\frac{2\pi}{W}\right)$

$X(j\omega) = \sum_{k=-\infty}^{\infty} \delta(\omega - kW)$

Exercise: Plot $x(t)$ $X(j\omega)$ by yourselves