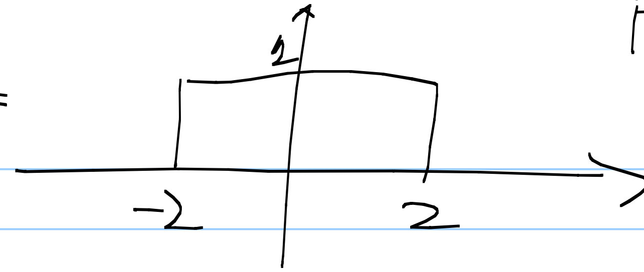
Another example

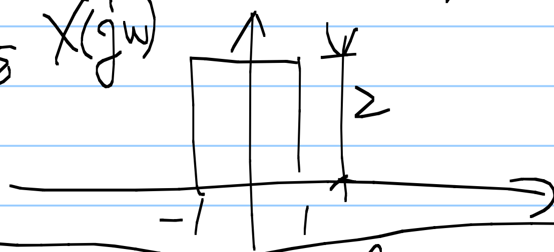
$Y(j\omega) =$



Knowing $y(t) = x(t)\cos(t)$

Find $X(j\omega)$ & $x(t)$

Ans $X(j\omega)$



$x(t) = \dfrac{2\sin(t)}{\pi t}$ ※

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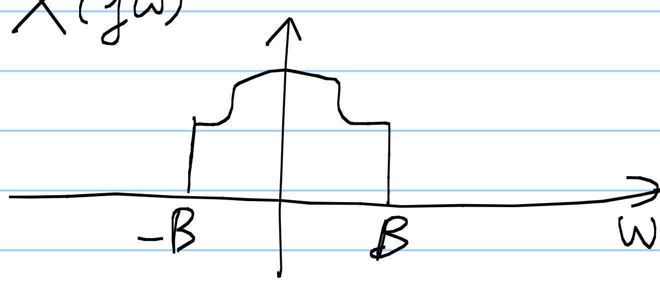
* An example of joint application of the multiplication / freq-shift & the convolution properties.

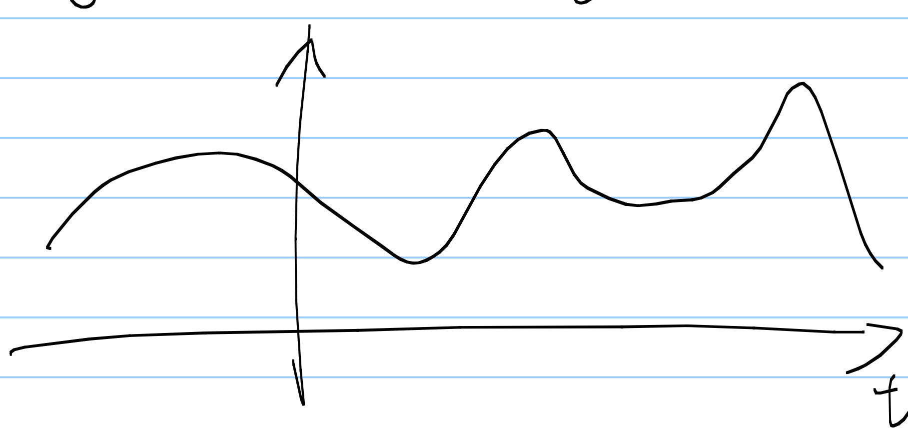
Suppose our original signal has a spectrum

$X(j\omega)$



Say $B = 20\,kHz$. Music signals.

$x(t)$



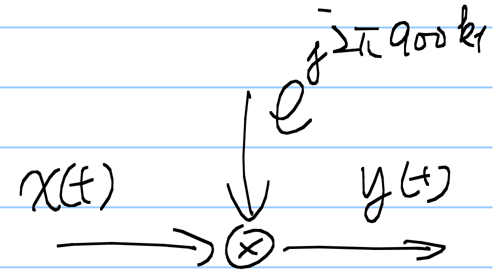
In physics, we know < 20 kHz signal cannot travel very far, but 900kHz signals travel much farther.

How to transmit $x(t)$ through a long distance?

Ans: MODULATION

Consider Method 1:

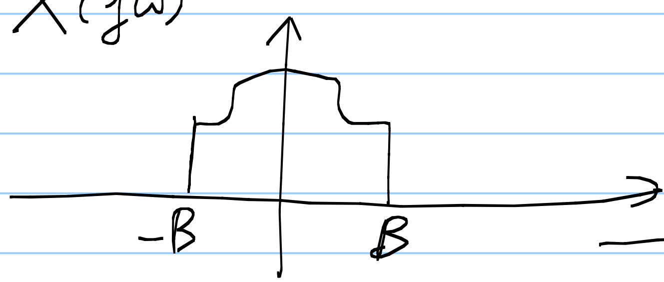
$$y(t) = x(t)\, e^{j2\pi\, 900k t}$$



Originally

New

X(jw)



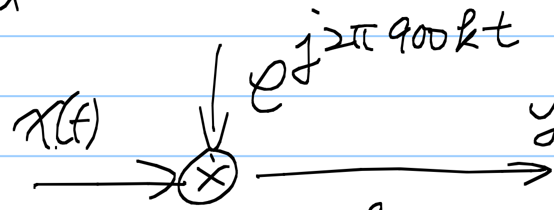
(base band)

Now the freq is shifted to a new freq band (pass band)

⇒ We can transmit the same content (The same shape of the freq spectrum) much further away.

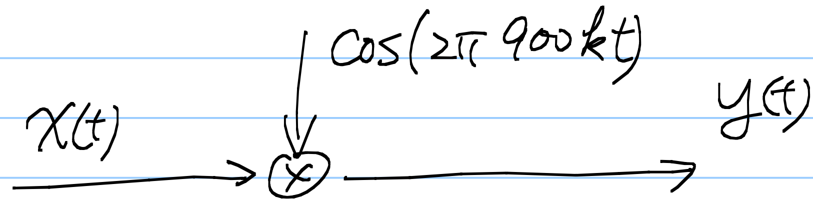
⇒ We say $x(t)$ has been modulated to the 900 kHz bandwidth.

The problem of the above "modulation method":



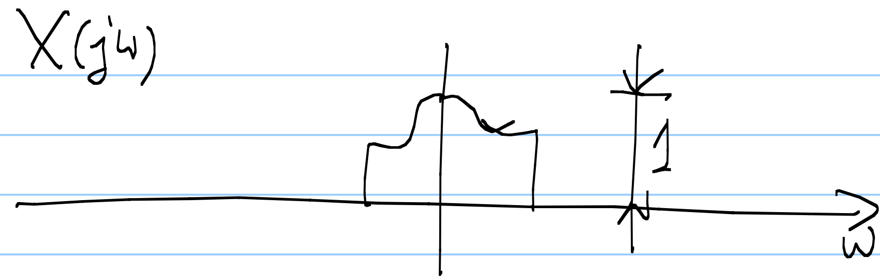
is $e^{j2\pi 900kHz}$ contains imaginary parts, which is not feasible.

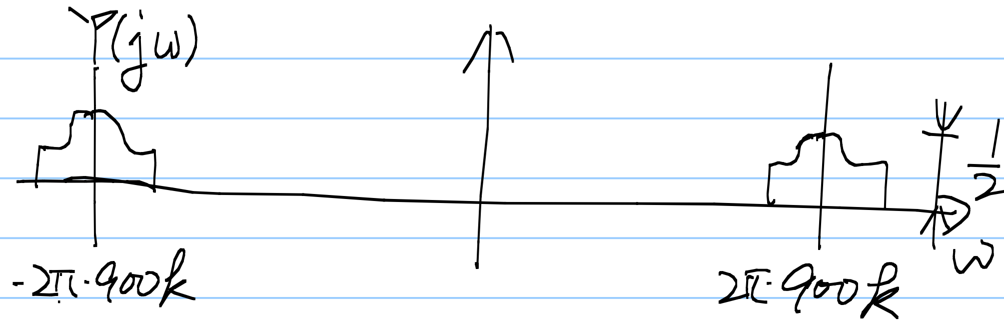
Modulation #2



$$y(t) = x(t)\cos(2\pi(900k)t)$$

Q: What is the "freq spectrum" of $y(t)$?





The spectrum of $x(t)$ has been split & $\frac{1}{2}$ of it is shifted to the right. & $\frac{1}{2}$ of it is shifted to the left

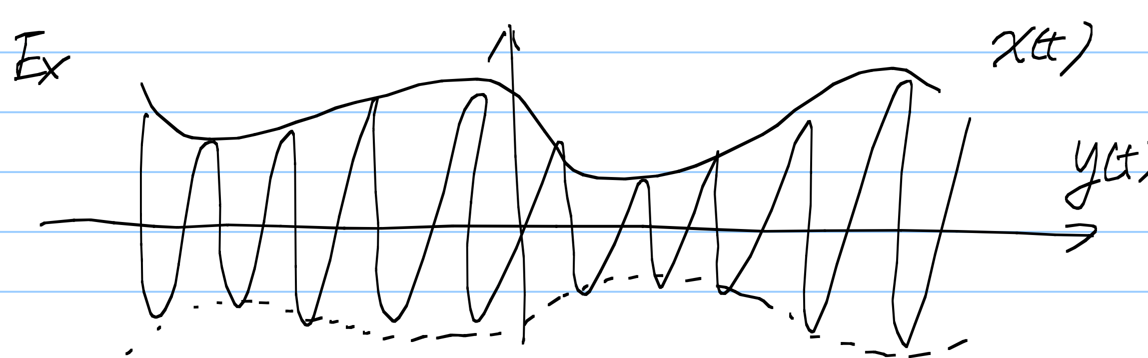
Now the $y(t)$ does not use any "base band freq". Therefore, $y(t)$ can be transmitted very far.

If $x(t)$ is a "slow varying" signal, then

$$y(t) = x(t)\cos(\omega_0 t)$$

can be thought of as a modulation of the amplitude of $\cos(\omega_0 t)$

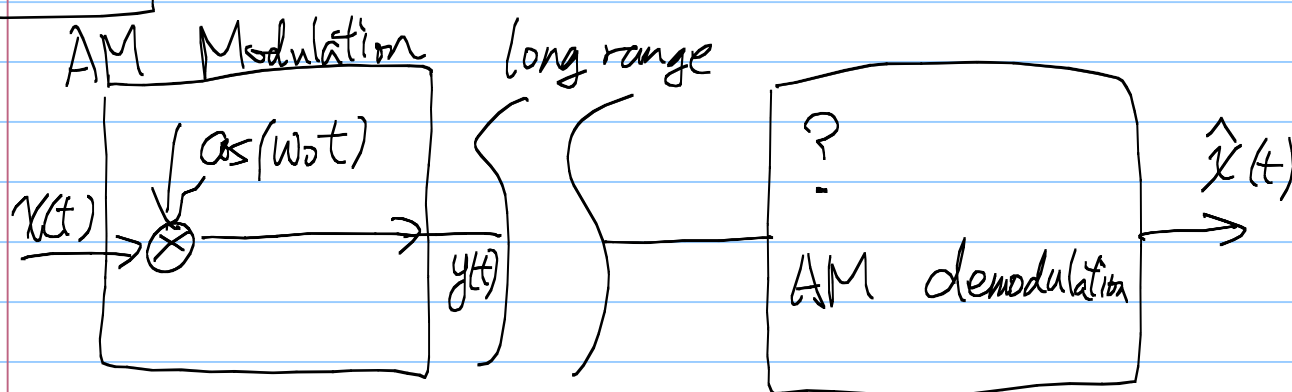
We use ω_0 for a general pass-band/<u>carrier freq</u>.



We thus say this method of modulation $y(t) = x(t)\cos(\omega_0 t)$ is Amplitude Modulation

In an AM radio, ω_0 ranges from 500 kHz to 1.6 MHz.

$\boxed{\text{AM}}$



We will talk more about AM in Chapter 8.

$\times$ 2nd Application of the multiplication property

Computing FT using the product form

Example $x(t) = \dfrac{\sin t \sin \frac{t}{2}}{\pi t^2}$

Find $X(j\omega)$

Ans: We notice that

$$x(t) = \pi\left(\frac{\sin t}{\pi t}\right)\left(\frac{\sin(t/2)}{\pi t}\right)$$

$$\Rightarrow X(j\omega) = \frac{1}{2\pi} \times \pi \,\mathcal{F}\left(\frac{\sin t}{\pi t}\right) * \mathcal{F}\left(\frac{\sin(t/2)}{\pi t}\right)$$