④ The multiplication property

$$x(t) \longleftrightarrow X(j\omega)$$

$$y(t) \longleftrightarrow Y(j\omega)$$

Convolution

$$x(t) * y(t) \longleftrightarrow X(j\omega) \cdot Y(j\omega)$$

Multiplication

$$z(t) = x(t) \cdot y(t) \longleftrightarrow Z(j\omega) = \frac{1}{2\pi} X(j\omega) * Y(j\omega)$$

Namely

$$Z(j\omega) = \left(\frac{1}{2\pi}\right) \int_{s=-\infty}^{\infty} X(js) \cdot Y(j(\omega - s))\, ds$$

Multiplication in the time domain

$\equiv$ Convolution in the freq domain (with $\frac{1}{2\pi}$ Scaling)

Example:

$y(t) = x(t) \cdot e^{j\omega_0 t}$ Find $Y(j\omega)$ in terms of $X(j\omega)$

Ans: $$Y(j\omega) = \frac{1}{2\pi} \left(X(j\omega) * \mathcal{F}\left(e^{j\omega_0 t}\right) \right)$$

$$= \frac{1}{2\pi} \left(X(j\omega) * \left(2\pi \delta(\omega - \omega_0) \right) \right)$$

$$= X(j\omega) * \delta(\omega - \omega_0)$$

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$$= \int_{s=-\infty}^{\infty} X(js) \cdot \delta((\omega - s) - \omega_0)\, ds$$

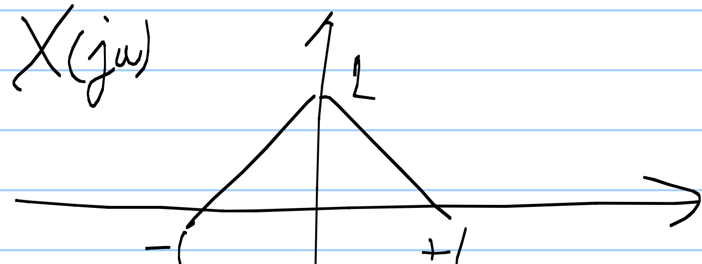
$$= \int_{s=-\infty}^{\infty} X(js) \cdot \delta((\omega - \omega_0) - s)\, ds$$

$$= X(j(\omega - \omega_0))$$

Convolution of a shifted delta
$\equiv$ direct shift of the original signal.

Example: $y(t) = x(t) \cdot \cos(\omega_0 t)$

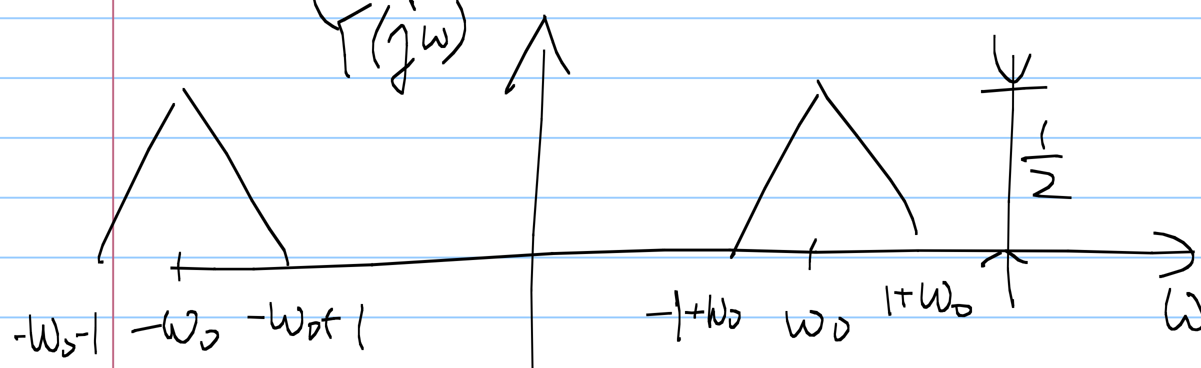
$X(j\omega)$



Q: Find $Y(j\omega)$.

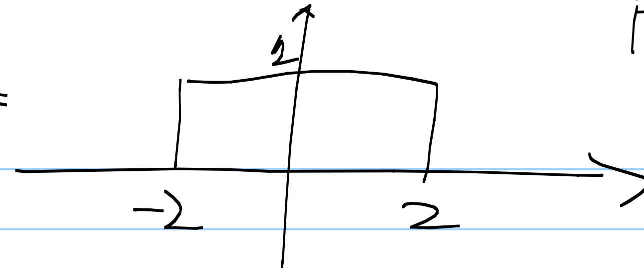
Ans: $y(t) = x(t) \cdot \frac{1}{2} e^{j\omega_0 t} + x(t) \cdot \frac{1}{2} e^{-j\omega_0 t}$

$Y(j\omega)$



$\boxed{\text{Another example}}$

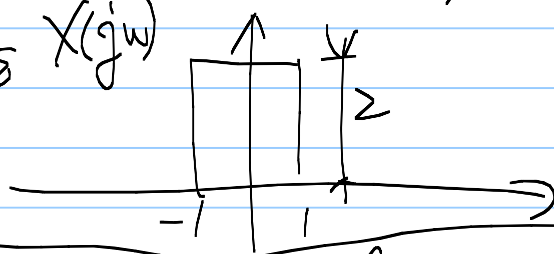
$Y(j\omega) =$



Knowing $y(t) = x(t)\cos(t)$

Find $X(j\omega)$ & $x(t)$

Ans $X(j\omega)$

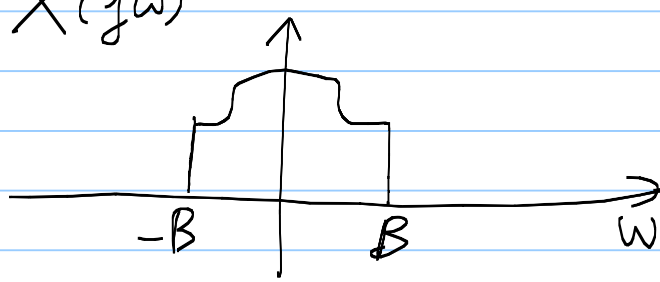


$$x(t) = \frac{2\sin(t)}{\pi t}$$ ※

* An example of joint application of the multiplication / freq-shift & the convolution properties.

Suppose our original signal has a spectrum

$X(j\omega)$



Say $B = 20\,kHz$. Music signals.

$x(t)$