* Other Applications of the convolution property:

1. Characterizing / Identifying LTI systems

Recall: In the past, we record h(t) by sending input $\delta(t)$. Then we compute $H(j\omega)$

An alternative way is to find $H(j\omega)$ directly

Namely, arbitrarily choose $x(t)$ and send it through the sys.

$$x(t) \longrightarrow \boxed{\text{unknown LTI}} \xrightarrow{\;y(t)\;} \text{Record } y(t)$$

Find $X(j\omega)$ & $Y(j\omega)$ by F.T, which can be done by computer.

$$\because \; Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$\therefore \; H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

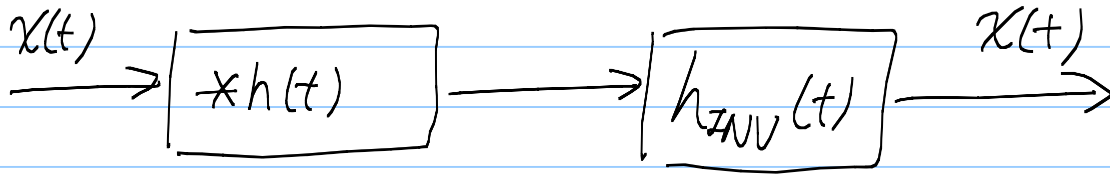
$$\Rightarrow h(t) = \mathcal{F}^{-1}(H(j\omega))$$

$\Rightarrow$ We do not need to feed an impuse signal to an LTI system.

Note that an "impulse" is very hard to generate since it has ∞ amplitude.

2. Inverting LTI sys.

Recall: if a sys is invertible



$\Rightarrow$ The impulse response of the concatenated system is

$$h(t) * h_{INV}(t) = \delta(t)$$

Q: $\mathcal{F}(\delta(t)) = ?$

Ans: 1

$$\Leftrightarrow H(j\omega) \cdot H_{INV}(j\omega) = \mathcal{F}(\delta(t)) = 1$$

$$\Rightarrow H_{INV}(j\omega) = \frac{1}{H(j\omega)}$$

δ in time $\Leftrightarrow$ horizontal in freq.

In summary, given $h(t)$, the $h_{INV}(t)$ can be found as follows.

$$h(t) \longrightarrow H(j\omega) \longrightarrow H_{INV}(j\omega) = \frac{1}{H(j\omega)}$$

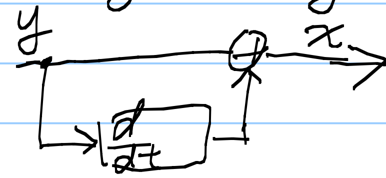
$$\longrightarrow h_{INV}(t) = \mathcal{F}^{-1}(H_{INV}(j\omega))$$

Example: $h(t) = e^{-t}u(t)$, find the inverse system

Sol'n: $H(j\omega) = \frac{1}{1+j\omega} \Rightarrow H_{INV}(j\omega) = 1 + j\omega$

$$X(j\omega) = (1+j\omega)\, Y(j\omega)$$

$$x(t) = y(t) + \frac{d}{dt} y(t)$$



P.129 * An LTI sys. is invertible if $H(j\omega) \neq 0$ for all ω.

2. Using "convolution in time $\equiv$ multiplication in freq" to invert an LTI system.

Example: Is $y(t) = \int_{t-T_1}^{t+T_1} x(s)\,ds$ invertible?

Ans: $h(t) = \int_{t-T_1}^{t+T_1} \delta(s)\,ds$

$$= u(t+T_1) - u(t-T_1)$$

$$\Rightarrow H(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

Can we find

$$H_{INV}(j\omega) = \frac{1}{H(j\omega)} \ ?$$

No. $\because$ Some ω values make $H(j\omega) = 0$

$\frac{1}{0}$ does not exist.

Example: $h(t) = \frac{\sin(Wt)}{\pi t}$ for some $W > 0$

Is this system invertible?

Ans: $H(j\omega) = u(\omega + W) - u(\omega - W)$

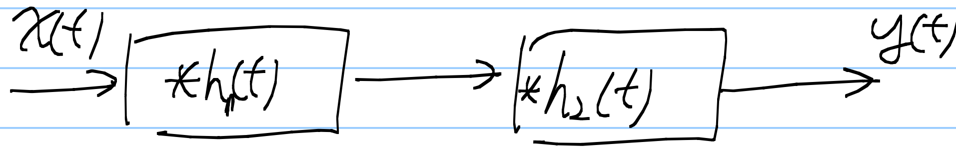
Can we find

$$H_{INV}(j\omega) = \frac{1}{H(j\omega)} \ ?$$

No. $\because$ Some ω values make $H(j\omega) = 0$

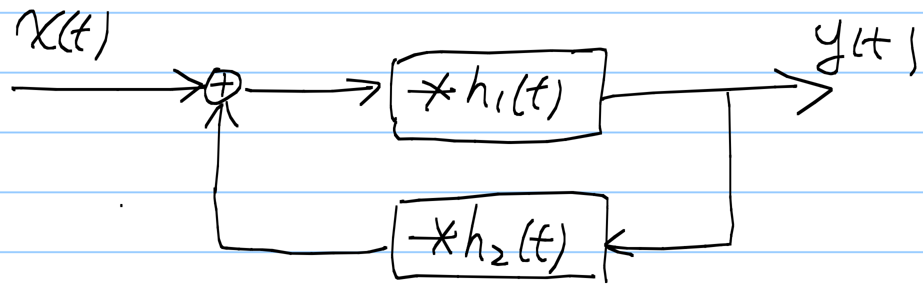
$\frac{1}{0}$ does not exist.

3. Interconnection of LTI sys.
(similar to Laplace Transform)



$$X(j\omega)\cdot \underbrace{H_1(j\omega)\cdot H_2(j\omega)}\;=\;Y(j\omega)$$

New $H(j\omega)$ for the big system



Like you have learned in Laplace transform

$$\Rightarrow Y(j\omega)=H_1(j\omega)\cdot\left(X(j\omega)+H_2(j\omega)\cdot Y(j\omega)\right)$$

$$\Rightarrow Y(j\omega)=\boxed{\frac{H_1(j\omega)}{1-H_1(j\omega)H_2(j\omega)}}\,X(j\omega)$$

New $H(j\omega)$ freq response.

4. Freq-based manipulation of the signal

Example:

An ideal low-pass filter (LPF) will ① keep any freq component within $|\omega| < W$ intact (also known as the bandwidth of a LPF)
② Completely supress any freq component outside (i.e $|\omega| > W$)

Since the output is

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$\Rightarrow H_{Ideal,LPF}(j\omega) = \begin{cases} 1 & \text{if } |\omega| < W \\ 0 & \text{otherwise} \end{cases}$$

So if we can design an LTI system with $h(t)$ satisfying $H(j\omega) = H_{Ideal,LPF}(\omega)$, then we have the ideal LPF

$$\Rightarrow h(t) = \mathcal{F}^{-1}\big(U(\omega+W) - U(\omega-W)\big)$$

$$= \frac{\sin Wt}{\pi t} \qquad \text{Example 4.5}$$