* Properties of CT FT. (Sec 4.3 of the textbook.)

① Linearity

$$a x(t) + b y(t) \xleftrightarrow{F.T} a X(j\omega) + b Y(j\omega)$$

② Time-shift

$$x(t) \xleftrightarrow{F.T} X(j\omega)$$

$$y(t) = x(t-t_0) \longleftrightarrow Y(j\omega) = X(j\omega) e^{-j\omega t_0}$$

pf: $y(t) = x(t-t_0) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(j\omega) \cdot e^{j\omega(t-t_0)} d\omega$

$$= \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} \underbrace{\left(X(j\omega) e^{-j\omega t_0}\right)}_{\text{must by } Y(j\omega)} e^{j\omega t} d\omega$$

③ Freq-shift

$$x(t) \xleftrightarrow{F.T} X(j\omega)$$

$$y(t) = x(t) e^{j\omega_0 t} \longleftrightarrow Y(j\omega) = X(j(\omega - \omega_0))$$

④ Time-Reversal ≡ Freq Reversal

$$y(t) = x(-t) \longleftrightarrow Y(j\omega) = X(-j\omega)$$

⑤ Time-Scaling: for some $a > 0$

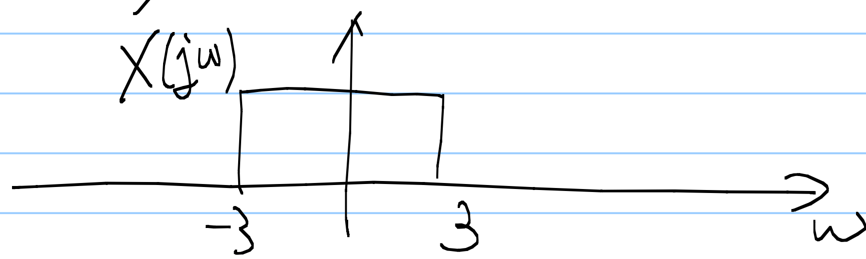
$$y(t) = x(at) \longleftrightarrow Y(j\omega) = \frac{1}{a} X\left(\frac{j\omega}{a}\right)$$

* An example about the freq-shift property.

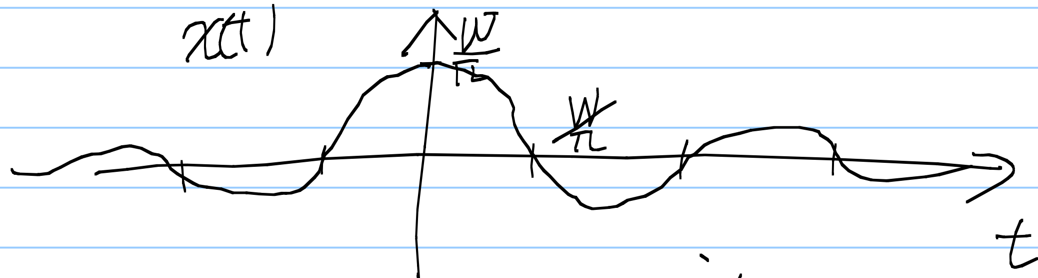
Example: $X(j\omega) = u(\omega+3) - u(\omega-3)$ $\boxed{P.12}$

(HW9 Q78): Find $x(t)$ & Plot it.

Ans: By Text Example 4.5




$$\Rightarrow x(t) = \frac{\sin Wt}{\pi t}$$

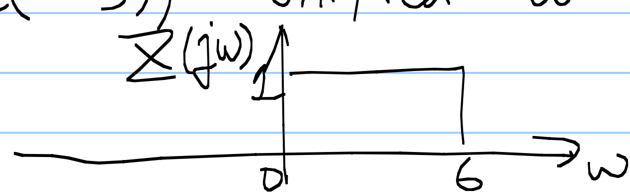



$z(t) = x(t)\,e^{j3t}$ Q: Find the F.T of $\underline{\underline{z(t)}}$.

Ans: By the freq-shift property

$$z(t) = x(t)\cdot e^{j\omega_0 t} \longleftrightarrow Z(j\omega) = X(j(\omega-\omega_0))$$

$\Rightarrow Z(j\omega) = X(j(\omega-3))$ shifted to the right. $\Rightarrow$



⑥ Differentiation

$$y(t)=\frac{dx(t)}{dt} \longleftrightarrow Y(j\omega)=j\omega X(j\omega)$$

pf. $y(t)=\frac{d}{dt}\frac{1}{2\pi}\int X(j\omega)e^{j\omega t}d\omega$

$$=\frac{1}{2\pi}\int X(j\omega)\frac{d}{dt}e^{j\omega t}d\omega$$

$$=\frac{1}{2\pi}\int \underline{X(j\omega)\cdot j\omega}\, e^{j\omega t}d\omega$$

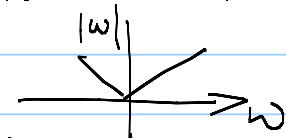
↪ must be $Y(j\omega)$

$$x(t) \rightarrow \boxed{\text{diff}} \rightarrow y(t)$$

⇒ Differentiation is a high-pass filter

∵ High freq components are amplified

by $|j\omega|=|\omega|$




Slow movement $\xrightarrow{\text{diff}}$ small values

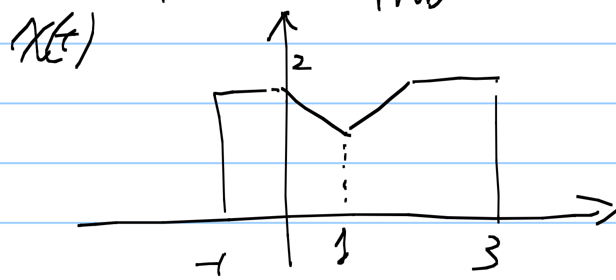
fast movement ⟶ large values

⑦ Parseval's Relationship (Law of energy conservation

$$\int_{t=-\infty}^{\infty}|x(t)|^2dt = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty}|X(j\omega)|^2d\omega$$

Total energy in time      Total energy in freq.

Example: Prob 4.25 (a, b, c, e)




(a) Find $\measuredangle X(jw)$

Ans: Consider $y(t)$

$\Rightarrow x(t) = y(t-1) \quad X(jw) = Y(jw) \cdot e^{-jw \cdot 1}$

* We use the property: The FT of an even signal has zero imaginary part.

$\Rightarrow \because y(t)$ is even $\therefore Y(jw)$ is real $\quad \measuredangle Y(jw) = 0$

$\Rightarrow \measuredangle X(jw) = \measuredangle Y(jw) + \measuredangle e^{-jw} = -w$ #

(b) Find $X(j0)$

Ans: $X(jw) = \int x(t) e^{-jwt} dt$

$$\Rightarrow X(j \cdot 0) = \int_{-\infty}^{\infty} x(t) \cdot e^{-jw \cdot 0} dt = 7$$

(c) Find $\int_{-\infty}^{\infty} X(jw) dw$

Ans: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) e^{jwt} dw$

$$\Rightarrow \int_{-\infty}^{\infty} X(jw) dw = 2\pi \, x(0) = 4\pi.$$

(e) Find $\int_{-\infty}^{\infty} |X(jw)|^2 dw$

Ans: $\int_{-\infty}^{\infty} |X(jw)|^2 dw = 2\pi \cdot \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{76\pi}{3}$