* Text Example 4.4

$$x(t) = \begin{cases} 1 & \text{if } |t| < T_1 \\ 0 & \text{otherwise} \end{cases}$$

Find $X(jw)$

Ans: Direct Computation

$$X(jw) = \int_{t=-\infty}^{\infty} x(t) e^{-jwt} dt$$

$$= \int_{-T_1}^{T_1} 1 \cdot e^{-jwt} dt$$

$$= \frac{1}{-jw} \left(e^{-jwT_1} - e^{jwT_1} \right)$$

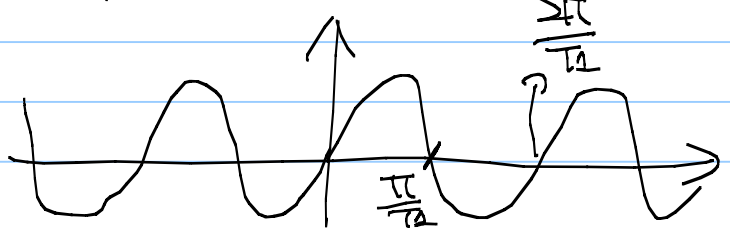
↓ exercise by Euler's formula

$$= \frac{2\sin(wT_1)}{w}$$

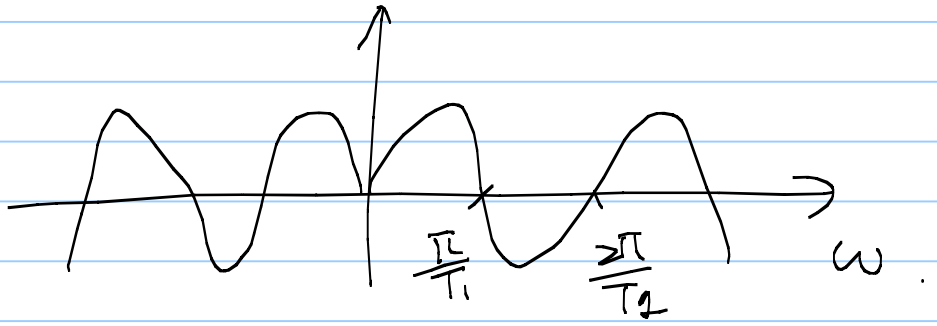
A function of w only.

Q: How to plot $X(jw)$?

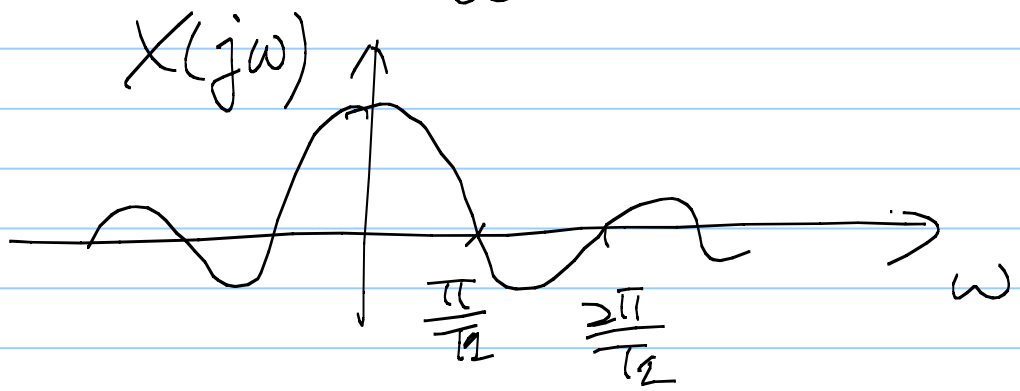
Ans: Step 1. Start from $\sin(wT_1)$



Step 2: plot $\frac{\sin(\omega T_1)}{\text{sign}(\omega)}$



Step 3: plot $\frac{2\sin(\omega T_1)}{\omega}$



Q: How high is the main lobe?

Ans: We use the fact that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$$\frac{2\sin(\omega T_1)}{\omega} = 2T_1 \cdot \underbrace{\frac{\sin(\omega T_1)}{\omega T_1}}_{\to 1 \text{ when } \omega \to 0.}$$

$$\Rightarrow \lim_{\omega \to 0} \frac{2\sin(\omega T_1)}{\omega} = 2T_1 \text{ the height of}$$

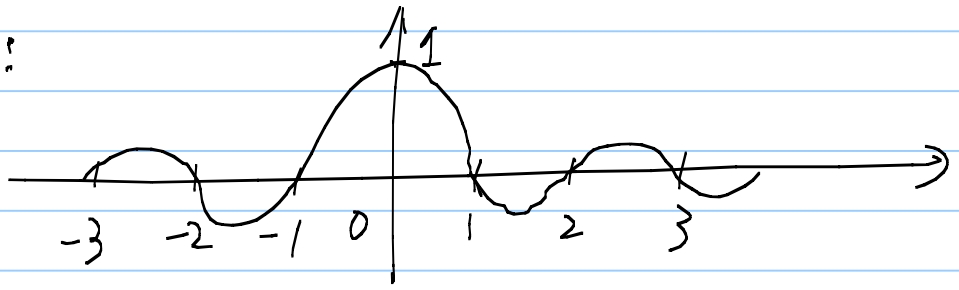
the main lobe.

* In the textbook, a "sinc" function is introduced:

$$\text{sinc}(\theta) = \frac{\sin \pi\theta}{\pi\theta}$$

Q: Plot $\text{sinc}(\theta)$ vs. θ ?

Ans:



* Alternatively, we can write our previous answer as

$$X(jw) = \frac{2\sin(wT_1)}{w}$$

$$= \frac{2T_1 \sin\left(\pi \cdot \frac{wT_1}{\pi}\right)}{\pi\left(\frac{wT_1}{\pi}\right)}$$

$$= 2T_1 \cdot \text{sinc}\left(\frac{wT_1}{\pi}\right) \#$$

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* Text Example 4.5

Given $X(jw) = \begin{cases} 1 & \text{if } |w| \le W \\ 0 & \text{if } |w| > W \end{cases}$

Find $x(t)$.

Ans: By direct computation

$$x(t) = \frac{1}{2\pi}\int_{w=-\infty}^{\infty} X(jw)\, e^{jwt}\, dw$$

$$= \frac{1}{2\pi}\int_{-\bar{W}}^{\bar{W}} 1 \cdot e^{jwt}\, dw$$

$$= \frac{1}{2\pi} \times \frac{1}{jt}\left(e^{j\bar{W}t} - e^{-j\bar{W}t}\right)$$

$$= \frac{1}{\pi t} \times \left(\sin(\bar{W}t)\right) \quad *$$

or equivalently

$$= \frac{\bar{W}}{\pi} \times \frac{\sin\left(\pi \cdot (\frac{\bar{W}}{\pi} t)\right)}{\pi \cdot (\frac{\bar{W}t}{\pi})}$$

$$= \frac{\bar{W}}{\pi} \cdot \operatorname{sinc}\left(\frac{\bar{W} \cdot t}{\pi}\right)$$

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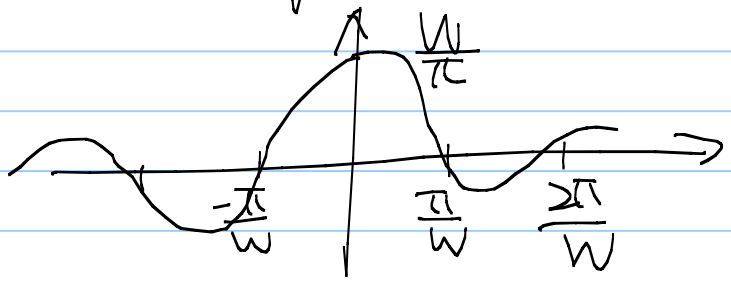

Step 1: Find the crossing points $(\sin(\,) = 0)$

Step 2: Find the height of the main lobe

using $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$ $\quad \frac{\bar{W}}{\pi} \cdot \frac{\sin(\bar{W}t)}{\bar{W}t}$

* The main lobe is twice as wide as the side lobe.