⑤ Difference

$$x[n] \xleftrightarrow{F.S} a_k, \frac{2\pi}{N}$$

$$y[n] = x[n] - x[n-1]$$

$$\longleftrightarrow b_k = a_k - a_k e^{-jk\frac{2\pi}{N}\cdot 1}$$

$$= a_k\left(1 - e^{-jk\frac{2\pi}{N}}\right), \quad \frac{2\pi}{N}$$

⑥ Parseval's relationship (Power conservation Law)

$$\frac{1}{N}\sum_{n\in\langle N\rangle} |x[n]|^2 = \sum_{k=0}^{N-1} |a_k|^2$$

avg power of $x[n]$

sum of avg power of each HRCE test signals.

HW8Q68 Prob 3.30 (a, b)

$$x[n] = 1 + \cos\left(\frac{2\pi}{6}n\right) \qquad y[n] = \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right)$$

Q: Find the DTFS of $x[n]$ & $y[n]$

Ans: ① $N = 6 \Longrightarrow \omega_0 = \frac{2\pi}{6}$

② $x[n] = 1 + \frac{1}{2}\left(e^{+j\frac{2\pi}{6}n} + e^{-j\frac{2\pi}{6}n}\right)$

p.101 $\Rightarrow a_0 = 1 \quad a_1 = \frac{1}{2} \quad a_{-1} = a_5 = \frac{1}{2}$

& $a_2, a_3, a_4 = 0.$

$$y[n] = \frac{1}{2j}\left(e^{+j\frac{2\pi}{6}n} \cdot e^{j\frac{\pi}{4}} - e^{-j\frac{2\pi}{6}n} \cdot e^{-j\frac{\pi}{4}}\right)$$

$\Rightarrow b_0 = 0, \quad b_1 = \frac{1}{2j} e^{j\frac{\pi}{4}}, \quad b_{-1} = b_5 = \frac{1}{2j} e^{-j\frac{\pi}{4}}$

$b_2, b_3, b_4 = 0$

HW8Q69

Q: $z[n] = x[n] \cdot y[n]$. Find the FS of $z[n]$

Ans: $N = 6$.

$$c_k = \sum_{h=0}^{5} a_h b_{k-h} = \sum_{h=0}^{5} b_h a_{k-h}$$

$$= b_1 a_{k-1} + b_5 a_{k-5}$$

$\Rightarrow c_0 = \frac{1}{2j} e^{j\frac{\pi}{4}} \cdot \left(\frac{1}{2}\right) + \frac{1}{2j} e^{-j\frac{\pi}{4}} \cdot \left(\frac{1}{2}\right)$

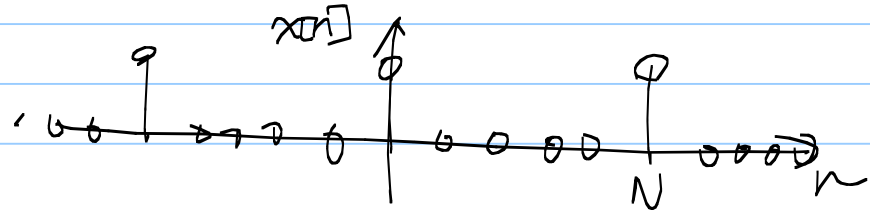
$c_1 = \frac{1}{2j} e^{j\frac{\pi}{4}} \cdot 1, \quad c_2 = \frac{1}{2j} e^{j\frac{\pi}{4}} \cdot \frac{1}{2}, \quad c_3 = 0, \quad c_4 = \frac{1}{2j} e^{-j\frac{\pi}{4}} \cdot \frac{1}{2}, \quad c_5 = \frac{1}{2j} e^{-j\frac{\pi}{4}} \cdot 1$

Example: For a given

$$x[n] = \sum_{m=-\infty}^{\infty} \delta[n - mN]$$

Plot $x[n]$ & Find the F.S.

Ans:

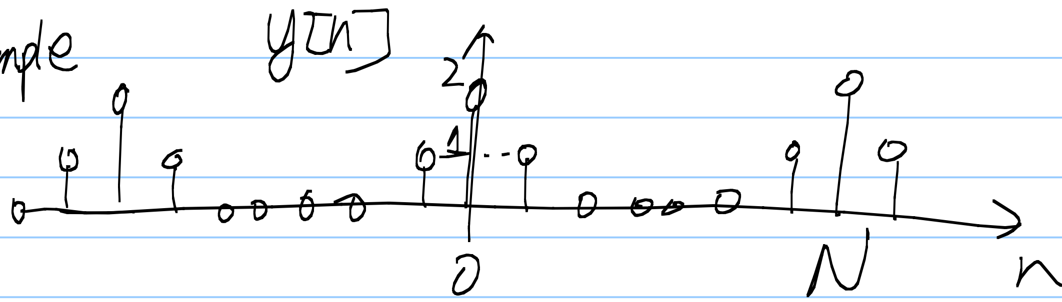


Ans: ① N

② $$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\frac{2\pi}{N}n}$$

$$= \frac{1}{N} \times 1 \times e^{-j\times 0} = \frac{1}{N} \quad \text{for all } k = 0, \cdots, N-1$$

Example



Find the F.S of $y[n]$

Ans: ① N

② Solution #1: direct computation

Solution #2:

$$y[n] = 2x[n] + x[n-1] + x[n+1]$$

$$\Rightarrow b_k = a_k \cdot \left(2 + e^{-jk\frac{2\pi}{N}\cdot 1} + e^{-jk\frac{2\pi}{N}\cdot(-1)}\right)$$

$$= \frac{1}{N}\cdot\left(2 + 2\cos\left(k\frac{2\pi}{N}\right)\right)$$