* Watch Video 3.6

* DT Fourier series representation

Subject : DT periodic signal $x[n]$ with period N

Representation :

Given N

$\boxed{\text{Synthesis formula}}$ $x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$

$\boxed{\text{Analysis formula}}$

$$a_k = \frac{1}{N} \sum_{n \in \langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}, \quad k=0, \cdots, N-1$$

* Two methods of finding a_k : ① Inspection & ② direct computation.

* An easy way to remember :

a_{-N} a_{-N+1} $\cdots$ a_{-1}

$\rightarrow$ a_0 a_1 a_2 $\cdots$ a_{N-1}

a_N, a_{N+1}, $\cdots$ a_{2N-1}

* Properties of DT FS

Selected properties (See Table 3.2 p.221 for a detailed list of properties)

1. Lineariy

$x[n] \xleftrightarrow{F.S.} a_k, \frac{2\pi}{N}$

$y[n] \xleftrightarrow{F.S.} b_k, \frac{2\pi}{N}$

$z[n] = Ax[n] + By[n]$

$\xleftrightarrow{F.S.} C_k = Aa_k + Bb_k, \frac{2\pi}{N}$

Write down your own comparison to CT.FS.

2. Time-Shift

$x[n] \longleftrightarrow a_k, \frac{2\pi}{N}$

$y[n] = x[n-n_0]$

$\longleftrightarrow b_k = a_k e^{-jk\omega_0 n_0}, \frac{2\pi}{N}$

3. Time-Reversal

Comparison to CTFS

$$x[n] \xleftrightarrow{FS} a_k, \frac{2\pi}{N}$$

$$y[n] = x[-n] \xleftrightarrow{FS} b_k = a_{-k} \quad \frac{2\pi}{N}$$

For example $b_0 = a_{-0} = a_0$

$b_1 = \underbrace{a_{-1}}_{\text{Not in the desired range.}} = a_{N-1}$

a_{-N}	a_{-N+1}		$\cdots$	a_{-1}
a_0	a_1	a_2	$\cdots$	a_{N-1}
a_N,	a_{N+1},		$\cdots$	a_{2N-1}

$$\therefore \; e^{j(-k)\frac{2\pi}{N}n} = e^{j(N-k)\frac{2\pi}{N}n}$$

So we often write

$$\begin{cases} b_0 = a_0 \\ b_k = a_{N-k} \quad \text{for } k = 1, \cdots, N-1 \end{cases}$$

4 Multiplication

Comparison to CT FS.

$$x[n] \xleftrightarrow{FS} a_k, \frac{2\pi}{N}$$

$$y[n] \xleftrightarrow{FS} b_k, \frac{2\pi}{N}$$

$$z[n] = x[n] \cdot y[n] \xleftrightarrow{FS} c_k, \frac{2\pi}{N}$$

Ans: $c_k = \sum_{h=0}^{N-1} a_h b_{k-h}$ Similar to the convolution sum, but is different as the summation is not over $\sum_{h=-\infty}^{\infty}$

We term it the "periodic/cyclic" convolution.

Example: Given $N=5$ &

$$x[n] \xleftrightarrow{F.S} a_0, \dots, a_4, \frac{2\pi}{5}$$

$$y[n] \xleftrightarrow{F.S} b_0, \dots, b_4, \frac{2\pi}{5}$$

$$z[n] = x[n] \cdot y[n] \longleftrightarrow c_0, \dots, c_4, \frac{2\pi}{5}$$

Q: Express c_2 in terms of a & b.

Ans: $c_2 = \sum_{h=0}^{4} a_h b_{2-h}$

$$= a_0 b_2 + a_1 b_1 + a_2 b_0 + \underbrace{a_3 b_{-1}}_{= a_3 b_4} + \underbrace{a_4 b_{-2}}_{= a_4 b_3}$$

$$= a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_4 + a_4 b_3$$