⑤ Differentiation

$$x(t) \longleftrightarrow a_k,\ \omega_0$$

$$y(t) = \frac{d}{dt} x(t) \quad \alpha > 0.$$

$$y(t) \longleftrightarrow b_k,\ \boxed{\underline{\underline{\omega_0}}}$$

Find the F.S of $y(t)$

Ans: ① ∵ differentiation does not change the period ⟹ $\underline{\underline{\quad\quad}} = \omega_0$.

$$\boxed{b_k = a_k \cdot (jk\omega_0)} \ \#$$

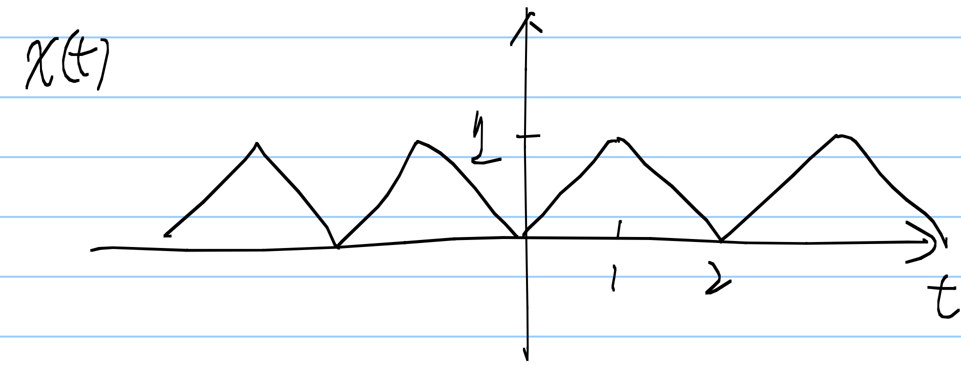
② by inspection

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Pf:

$$y(t) = \frac{d}{dt} x(t) = \sum_{k=-\infty}^{\infty} \underbrace{a_k \cdot (jk\omega_0)}_{\text{must be } b_k} e^{jk\omega_0 t}$$

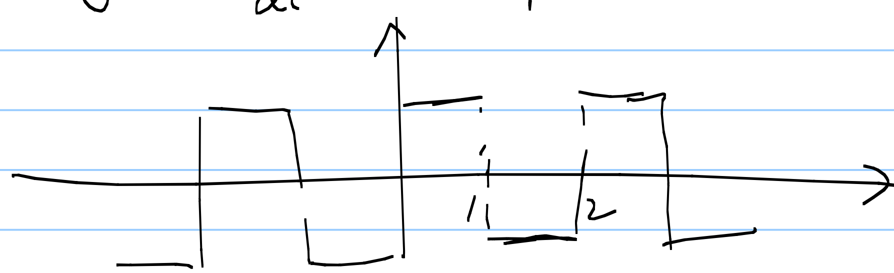
HW7Q63 Prob 3.24.



Q: F.S.

Ans: $\omega_0 = \frac{2\pi}{2} = \pi$.

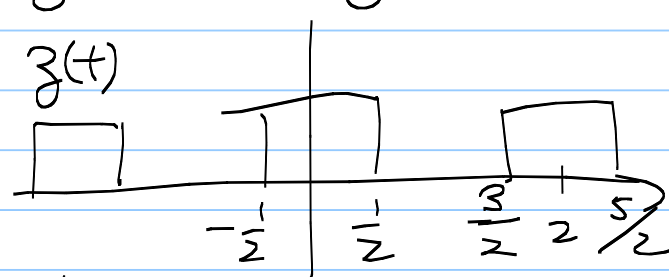
Sub-Q: $y(t) = \frac{d}{dt} x(t)$, plot $y(t)$



Q: F.S ~~of~~ $y(t)$.

Ans: Note that $y(t) = z(t-0.5) - z(t-1.5)$

where $z(t)$



by Example 3.5

$z(t)$ has $\quad C_0 = \frac{2 \cdot \frac{1}{2}}{2} = \frac{1}{2}$

$C_k = \frac{\sin(k\frac{2\pi}{2} \cdot \frac{1}{2})}{k\pi} = \frac{\sin(\frac{k\pi}{2})}{k\pi}$

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By linearity, the FS coeff of $y(t)$ is $b_k = C_k\left(e^{-jk\frac{2\pi}{2}\cdot\frac{1}{2}} - e^{-jk\frac{2\pi}{2}\cdot\frac{3}{2}}\right)$

& $\Rightarrow$ $b_0 = C_0(1-1) = 0$.

for those $k \neq 0$.

$$b_k = \frac{\sin(k\cdot\frac{\pi}{2})}{k\pi}\cdot\left(e^{-jk\frac{\pi}{2}} - e^{-jk\frac{3\pi}{2}}\right)$$

$$= \frac{\sin(k\cdot\frac{\pi}{2})}{k\pi}\cdot\left(e^{-jk\frac{\pi}{2}} - e^{+jk\frac{\pi}{2}}\right)$$

$$\because \left(e^{-jk\frac{3\pi}{2}}\right) = \left(e^{-j\frac{3\pi}{2}}\right)^k$$

$$= \left(e^{j\frac{\pi}{2}}\right)^k$$

$$= \frac{\sin(k\cdot\frac{\pi}{2})}{k\pi}\cdot\left(-2j\sin(k\cdot\frac{\pi}{2})\right)$$

On the other hand, since

$$y_k = \frac{d}{dt}x(t)$$

$$\Rightarrow b_k = a_k\cdot(jk\omega_0)$$

if $k \neq 0$ $\quad a_k = \frac{b_k}{jk\omega_0} = (-2)\frac{\sin^2(k\cdot\frac{\pi}{2})}{(k\pi)^2}$.

$a_0 = \frac{1}{2}$ by inspection.

⑥ Multiplication

Suppose

$$x(t) \xleftrightarrow{F.S} a_k, \quad w_0$$
$$y(t) \xleftrightarrow{F.S} b_k, \quad w_0$$

the same freq w_0

$$z(t) = x(t) \cdot y(t) \xleftrightarrow{F.S} c_k, \boxed{w_0}$$

Find the F.S. representation of $z(t)$.

Ans. ① ___ $= w_0$ ∵ multiplication does not change the freq.

② $\boxed{c_k = a_k * b_k}$

convolve

$$c_k = \sum_{h=-\infty}^{\infty} a_h \cdot b_{k-h}$$

pf: $z(t) = \left(\sum_{h=-\infty}^{\infty} a_h e^{jhw_0 t}\right)\left(\sum_{k'=-\infty}^{\infty} b_{k'} e^{jk' w_0 t}\right)$

$$= \sum_{k'=-\infty}^{\infty} \sum_{h=-\infty}^{\infty} a_h b_{k'} e^{j(h+k')w_0 t}$$

Choose $k = h + k'$

$$= \sum_{k=-\infty}^{\infty} \underbrace{\sum_{h=-\infty}^{\infty} a_h b_{k-h}}_{\text{must be } C_k} e^{j(k)w_0 t}$$

⑦ Parseval's Relationship

If

$$x(t) \longleftrightarrow a_k,$$

then $$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

Intuition, we first note that

$$\frac{1}{T}\int_T |e^{jkw_0 t}|^2 dt = 1.$$

$\Rightarrow$ The average power of a HRCE e^{jkw_0t} is 1.

$$\Rightarrow \frac{1}{T}\int |a_k \cdot e^{jkw_0t}|^2 dt = |a_k|^2$$

Since $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0t}$ contains many HRCEs, the avg power of $x(t)$ should be the sum of the avg powers of each HRCE.

* The Parseval's relationship is sometimes referred as the "<u>power conservation law.</u>"