③ Time reversal

$$x(t) \xleftrightarrow{F.S} a_k,\ \omega_0$$

$y(t) = x(-t)$

$$y(t) \xleftrightarrow{F.S} b_k,\ \boxed{\underline{\underline{\omega_0}}}.$$

Ans: $\underline{\underline{\qquad}} = \omega_0$ since time-reversal does not change ω_0.

$$\boxed{b_k = a_{-k}}$$

pf 1: Direct computation — p.203

pf 2: Inspection

$$y(t) = x(-t)$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0(-t)} = \sum_{k=-\infty}^{\infty} a_k e^{-jk\omega_0 t}.$$

Let $k' = -k$

$$= \sum_{k'=-\infty}^{\infty} \underline{\underline{a_{-k'}}}\, e^{jk'\omega_0 t}$$ → must be b_k.

④ Time scaling

$x(t) \longleftrightarrow a_k,\ \omega_0$

$y(t) = x(\alpha t) \quad \alpha > 0.$

$y(t) \longleftrightarrow b_k,\ \boxed{\alpha\omega_0}$

Ans: ∵ Time scaling changes the period

$\Rightarrow$ the new period is $\frac{T}{\alpha}$.

$\Rightarrow$ the new freq is $\omega = \frac{2\pi}{T/\alpha}$

$= \alpha \times \frac{2\pi}{T} = \alpha\omega_0$

$\boxed{b_k = a_k}$

pf 1: Direct computation — p.204

pf 2: $y(t) = x(\alpha t)$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0(\alpha t)}$$

$$= \sum_{k=-\infty}^{\infty} \underbrace{a_k}_{\text{must be } b_k} e^{jk\underbrace{(\alpha\omega_0)}_{\text{New freq}}t}$$

Remark 1: Time-scaling is the only property that involves <u>freq-change</u>

Remark 2: Two F.S. representations are the same only when

$$a_k = b_k \quad \text{and} \quad \underbrace{\omega_1 = \omega_2}_{\text{the freq}}$$

$\boxed{\text{Question for the teams}}$

Prove that "if $x(t)$ is an even signal, then $a_k = a_{-k}$ for all k."

Prove that "if $a_k = a_{-k}$ for all k, then $x(t)$ is an even signal".

Can you derive similar arguments for the case in which $x(t)$ is an odd signal?