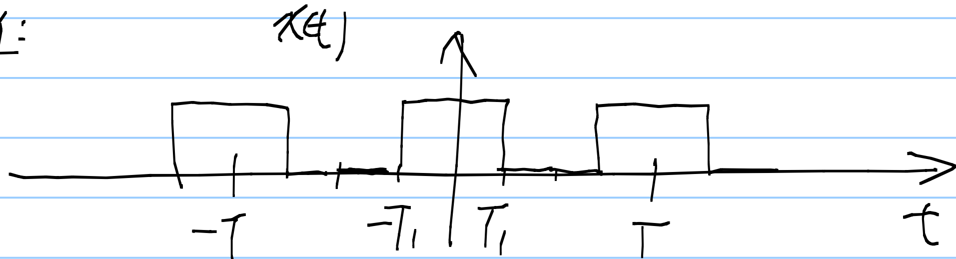
Method #2 Direct computation

Text Example 3.5 $x(t) = \begin{cases} 1 & \text{if } |t| < T_1 \\ 0 & \text{if } T_1 < |t| < \frac{T}{2} \\ \text{periodic with period } T \end{cases}$

Q_1: Plot $x(t)$

Q_2: Find its FS representation.

A_1:



A_2: $\omega_0 = \frac{2\pi}{T}$ (Find it freq first)

a_0 ($k=0$) usually needs to be computed separately

$$a_0 = \frac{1}{T}\int_T x(t)\, e^{-j \cdot 0 \cdot \frac{2\pi}{T} t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(t) \cdot 1\, dt$$

$$= \frac{1}{T}\int_{-T_1}^{T_1} 1 \times 1\, dt = \frac{2T_1}{T}$$ the DC component

For $k \neq 0$

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\frac{2\pi}{T}t}\, dt$$

$$= \frac{1}{T}\int_{-T_1}^{T_1} 1 \times e^{-jk\frac{2\pi}{T}t}\, dt$$

$$= \frac{1}{T}\,\frac{e^{-jk\frac{2\pi}{T}\times T_1} - e^{-jk\frac{2\pi}{T}(-T_1)}}{-jk\frac{2\pi}{T}}$$

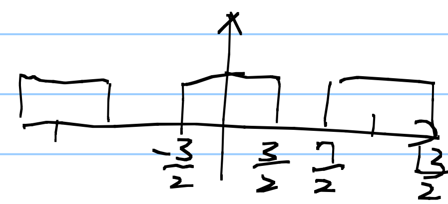
$$= \frac{\sin\left(k\frac{2\pi}{T}T_1\right)}{k\pi}$$

(Q: why a_0 needs to be considered separately?

Synthesis

$$x(t) = \frac{2T_1}{T} + \sum_{k=-\infty}^{\infty}\left(\frac{\sin\left(k\frac{2\pi}{T}T_1\right)}{k\pi}\right)e^{jk\frac{2\pi}{T}t}$$

Example $T=5$, $T_1 = \frac{3}{2}$,



Q: Find its FS representation.

Ans: $\omega_0 = \frac{2\pi}{5}$

$$\Rightarrow a_0 = \frac{2T_1}{T} = \frac{3}{5}$$

$$a_k = \frac{\sin\left(k\frac{2\pi}{5}\times\frac{3}{2}\right)}{k\pi}$$

$$= \frac{\sin\left(\frac{3k}{5}\pi\right)}{k\pi}$$

$a_1 = a_{-1} = 0.3027$

$a_2 = a_{-2} = -0.0935$

$a_3 = a_{-3} = -0.0624$

$a_4 = a_{-4} = 0.0757$

$$x(t) = a_0 + \left(a_1 e^{j\frac{2\pi}{5}t} + a_{-1} e^{-j\frac{2\pi}{5}t} \right) + \left(a_2 e^{j2\times\frac{2\pi}{5}t} + a_{-2} e^{j(-2)\cdot\frac{2\pi}{5}t} \right) + \cdots$$

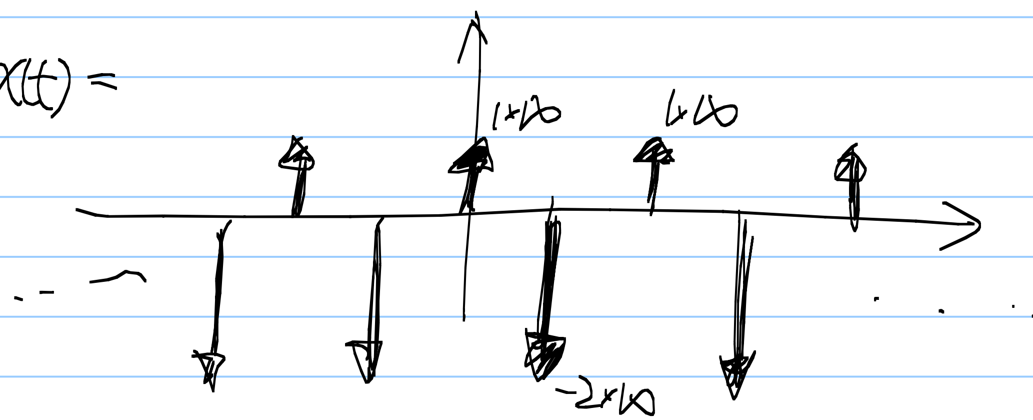
$$= a_0 + a_1\left(2\cos\left(\frac{2\pi}{5}t\right)\right) + a_2\times\left(2\cos\left(\frac{2\times2\pi}{5}t\right)\right) + \cdots$$

The summation of many cosine signals

* See the additional handout.

HW6Q56 Prob 3.22(a) — fig (d)

$x(t) =$



Q: $x(t)$ is continuous-time or discrete-time?

A = continuous-time

Q Find its FS representation.

Ans: ① Always find ω_0 first.

$T=2$, $\omega_0 = \frac{2\pi}{2} = \pi$

(2) $a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt$

$= \frac{1}{2}\int_{-0.5}^{1.5} \left(\delta(t) - 2\delta(t-1)\right) e^{-jk\omega_0 t} dt$

$= \frac{1}{2}\int_{-0.5}^{1.5} \delta(t) e^{-jk\omega_0 t} dt + \int_{-0.5}^{1.5} \delta(t-1) e^{-jk\omega_0 t} dt$

$= \frac{1}{2} \times \left(e^{-jk\omega_0 \times 0} - 2e^{-jk\omega_0 \times 1} \right)$

$= \frac{1}{2} - e^{-jk\pi}$

$= \frac{1}{2} - (-1)^k$

ex: $a_0 = \frac{1}{2} - 1 = -\frac{1}{2}$ the DC component.