Note Title 2/10/2010

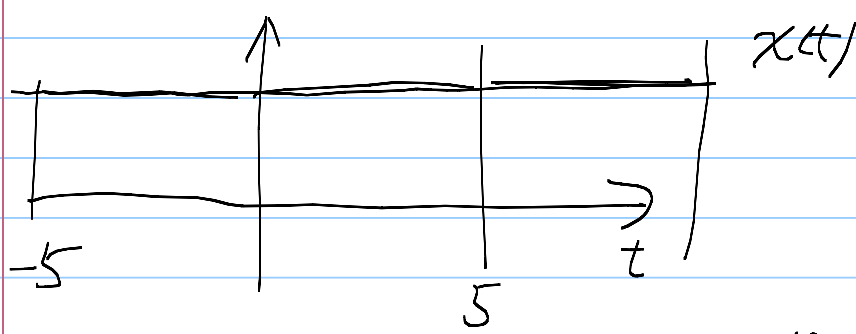
* Examples of computing the FS coefficients

Method #1: by inspection Method #2: by direct computation

Example 1: (Solved by inspection)

$x(t)$ is of period 5.

$$x(t) = \begin{cases} 3 & \text{if } 0 \le t < 5 \\ \& \ x(t) \text{ is periodic w. period } 5 \end{cases}$$



Q: Compute the FS coeff of $x(t)$.

Ans: $w_0 = \frac{2\pi}{5}$

$$x(t) = a_0 \times 1 + a_1 e^{j 1 \times \frac{2\pi}{5} t} + a_2 e^{j 2 \times \frac{2\pi}{5} t} + \ldots + a_{-1} e^{j(-1)\frac{2\pi}{5} t} + a_{-2} e^{j(-2)\frac{2\pi}{5} t} + \ldots$$

By inspection, $a_0 = 3$, the DC component.

No other oscillation $\Longrightarrow$ $a_k = 0$ for all $k \neq 0$.

Example 2: (Solved by inspection)

$x(t) = \cos(\frac{2\pi}{5}t) + 1$. Q: Find its FS representation.

Ans: Whenever we are asked to find a FS representation, we need to find both the fundamental freq ω_0 & the coeff a_k. (Only knowing a_k is not enough for writing down $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$)

① $T = \frac{2\pi}{2\pi/5} = 5 \Rightarrow \omega_0 = \frac{2\pi}{5}$ #

②

$$x(t) = a_0 \times 1 + a_1 e^{j1\times\frac{2\pi}{5}t} + a_2 e^{j2\times\frac{2\pi}{5}t} + \cdots + a_{-1} e^{j(-1)\frac{2\pi}{5}t} + a_{-2} e^{j(-2)\frac{2\pi}{5}t} + \cdots$$

by inspection $a_0 = 1$. Moreover since $\cos(\frac{2\pi}{5}t) = \frac{1}{2}(e^{j\frac{2\pi}{5}t} + e^{-j\frac{2\pi}{5}t}) \Rightarrow a_1 = \frac{1}{2}, a_{-1} = \frac{1}{2}$, All other $a_k = 0$.

Example 3: $x(t) = \sin(3t) + 3\cos(2t + \frac{\pi}{4})$

Find its FS representation

Ans: ① find its w_0.

$$T = LCM\left(\frac{2\pi}{3}, \frac{2\pi}{2}\right) \Rightarrow 2\pi$$

$$w_0 = \frac{2\pi}{2\pi} = 1$$

② We are interested in

$$x(t) = a_0 + \begin{matrix} a_1 e^{j1\times 1t} \\ a_{-1} e^{j(-1)\times 1\times t} \end{matrix} + \begin{matrix} a_2 e^{j2\times 1\times t} \\ a_{-2} e^{j(-2)\times 1\times t} \end{matrix} + \begin{matrix} a_3 e^{j3\times 1\times t} \\ a_{-3} e^{j(-3)\times 1\times t} \end{matrix}$$

By Euler's formula.

$$\sin(3t) = \frac{1}{2j}\left(e^{j3t} - e^{-j3t}\right)$$

$$3\cos(2t + \frac{\pi}{4}) = 3\times\left(\frac{1}{2}\left(e^{j(2t+\frac{\pi}{4})} + e^{-j(2t+\frac{\pi}{4})}\right)\right)$$

$$= \frac{3}{2}\times e^{j\frac{\pi}{4}} e^{j2t} + \frac{3}{2} e^{-j\frac{\pi}{4}} e^{-j2t}$$

$\Rightarrow$ By inspection $a_0 = 0$ $a_1 = a_{-1} = 0$

$$a_2 = \frac{3}{2} e^{j\frac{\pi}{4}} = \frac{3}{2}\left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)$$

$$a_{-2} = \frac{3}{2} e^{-j\frac{\pi}{4}} = \frac{3}{2}\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$$

$a_3 = \frac{1}{2j} = -\frac{j}{2}$ $a_{-3} = -\frac{1}{2j} = \frac{j}{2}$ all other $a_k = 0$