= The output of a LTI sys with special (classes of) input signals.

* When the input is a shifted impulse, the output is a shifted impulse respons.

* What if the input is

  a complex exponential $e^{j\omega t}$ or $e^{j\omega n}$?

$\boxed{CT}$ Q: For a given LTI system with impulse response $h(t)$

$$\delta(t) \xrightarrow{\quad} \boxed{LTI} \xrightarrow{\quad} h(t)$$

$$e^{j\omega t} \xrightarrow{\quad} \boxed{LTI} \xrightarrow{\quad} y(t) = ?$$

Find out the output $y(t)$.

Ans: $$y(t) = \int_{s=-\infty}^{\infty} e^{+j\omega s}\, h(t-s)\, ds$$

$$= \int_{s=-\infty}^{\infty} h(s)\, e^{+j\omega(t-s)}\, ds$$

$$= e^{j\omega t} \underbrace{\int_{-\infty}^{\infty} h(s)\, e^{-j\omega s}\, ds}_{\text{A value depends only on } \omega}$$

Denote that value by $H(j\omega)$.

$$\Rightarrow \boxed{y(t) = e^{j\omega t} \cdot H(j\omega)} \qquad \text{☆ } H(j\omega) = \int_{-\infty}^{\infty} h(s) e^{-j\omega s} ds$$

Example 1: when $h(t) = u(t) - u(t-1)$, Q: $H(j\omega) = ?$

Example 2: when $h(t) = e^{-t}u(t)$, Q: $H(j\omega) = ?$

☆ Note ~~that~~ $H(j\omega)$ may be a complex value, ex: $\frac{1-e^{-j\omega}}{j\omega}$ or $\frac{1}{1+j\omega}$

☆☆ For an LTI system, when the input is $x(t) = e^{j\omega t}$, the output is

$$y(t) = e^{j\omega t} \underbrace{H(j\omega)}_{\text{coefficient}}$$

$$= e^{j\omega t} \left( \underbrace{|H(j\omega)|}_{\text{amplification}} e^{j \overbrace{\measuredangle H(j\omega)}^{\text{phase shift}}} \right)$$

$y(t)$ is of the same freq $\omega$ as $x(t)$

The only change is its amplitude (amplified by $H(j\omega)$) & its phase (shifted by $\measuredangle H(j\omega)$)

Example:

$$\delta(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(t) = e^{-t}u(t)$$

Q: Find out the output $y_1(t)$ when the input is $x_1(t) = \cos(t)$
(similar to F11, Final, Q5)

Ans: Let us solve a different problem first.

Q: Find out the output $y(t)$ when the input is $x(t) = e^{j\omega t}$.

Ans: We know that

$$y(t) = e^{j\omega t} \cdot H(j\omega) \quad \text{and} \quad H(j\omega) = \int_{-\infty}^{\infty} h(s) e^{-j\omega s} ds$$

$$H(j\omega) = \int_0^{\infty} e^{-s} e^{-j\omega s} ds$$

$$= \frac{1}{1+j\omega} = \frac{1-j\omega}{1+\omega^2}$$

$\Rightarrow$ When $\omega = 1$, $x(t) = e^{j \cdot t} = \cos(t) + j\sin(t)$

$$y(t) = e^{jt}\left(\frac{1-j}{2}\right)$$

$$= e^{jt}\cdot\left(\frac{1}{\sqrt{2}}\, e^{j\left(-\frac{\pi}{4}\right)}\right)$$

$$= \frac{1}{\sqrt{2}}\cos\left(t-\frac{\pi}{4}\right) + \frac{1}{\sqrt{2}}\, j\sin\left(t-\frac{\pi}{4}\right)$$

$\Rightarrow$ When the input is $\cos(t)$ $(Re(x(t)))$
the output is $\frac{1}{\sqrt{2}}\cos\left(t-\frac{\pi}{4}\right)$ $(Re(y(t)))$

** Sinusoidal in $\longrightarrow$ $\boxed{\text{LTI}}$ $\longrightarrow$ Sinusoidal out with new amplitude & phase

Exercise:

Q: Find out the output $y_2(t)$
when the input is $x_2(t) = \cos(\sqrt{3}t)$

A: $y_2(t) = \frac{1}{2}\cos\left(\sqrt{3}t - \frac{\pi}{3}\right)$

$\because$ $\cos(t) \to \frac{1}{\sqrt{2}}\cos\left(t-\frac{\pi}{4}\right)$

$\cos(\sqrt{3}t) \to \frac{1}{2}\cos\left(\sqrt{3}t-\frac{\pi}{3}\right)$

$\Rightarrow$ Low freq part is stronger. You feel stronger bass in the music

* Watch Video 2.3.10