

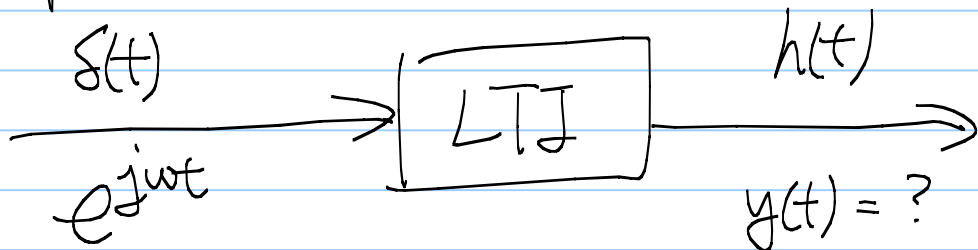
= The output of a LTI sys with special (classes of) input signals.

* When the input is a shifted impulse, the output is a shifted impulse response.

* What if the input is

CT a complex exponential $e^{j\omega t}$ or $e^{j\omega n}$?

Q: For a given LTI system with impulse response $h(t)$



Find out the output $y(t)$.

$$\text{Ans: } y(t) = \int_{s=-\infty}^{\infty} e^{+j\omega s} h(t-s) ds$$

$$= \int_{s=-\infty}^{\infty} h(s) e^{+j\omega(t-s)} ds$$

$$= e^{j\omega t} \underbrace{\int_{-\infty}^{\infty} h(s) e^{-j\omega s} ds}$$

A value depends only on ω

Denote that value by $\underline{H(j\omega)}$. p. 069

$$\Rightarrow \boxed{y(t) = e^{j\omega t} \cdot H(j\omega)} \quad \star H(j\omega) = \int_{-\infty}^{\infty} h(s) e^{-j\omega s} ds$$

Example 1: when $h(t) = u(t) - u(t-1)$, Q: $H(j\omega) = ?$

Example 2: when $h(t) = e^{-t}u(t)$, Q: $H(j\omega) = ?$

\star Note that $H(j\omega)$ may be a complex value, ex: $\frac{-e^{-j\omega}}{j\omega}$ or $\frac{1}{1+j\omega}$

$\star\star$ For an LTI system, when the input is $x(t) = e^{j\omega t}$, the output is

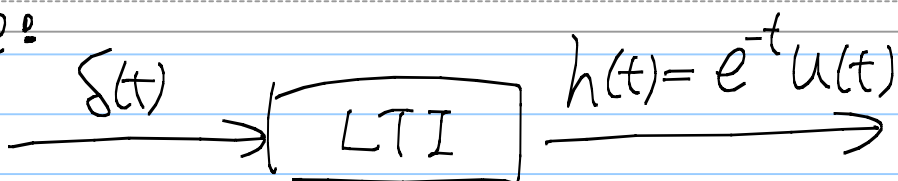
$$y(t) = e^{j\omega t} \underline{H(j\omega)} \rightarrow \text{coefficient}$$
$$= e^{j\omega t} \left(\underline{|H(j\omega)|} e^{j \underline{\angle H(j\omega)}} \right)$$

↓ amplification ↘ phase shift

$|y(t)|$ is of the same freq ω as $x(t)$

The only change is its amplitude (amplified by $H(j\omega)$) & its phase (shifted by $\angle H(j\omega)$)

Example:



Q: Find out the output $y_1(t)$
 when the input is $x_1(t) = \cos(t)$
 (similar to F11, Final, Q5)

Ans: Let us solve a different problem first.

Q: Find out the output $y(t)$
 when the input is $x(t) = e^{j\omega t}$.

Ans: We know that

$$y(t) = e^{j\omega t} \cdot H(j\omega) \quad \text{and} \quad H(j\omega) = \int_{-\infty}^{\infty} h(s) e^{-j\omega s} ds$$

$$H(j\omega) = \int_0^{\infty} e^{-s} e^{-j\omega s} ds$$

$$= \frac{1}{1+j\omega} = \frac{1-j\omega}{1+\omega^2}$$

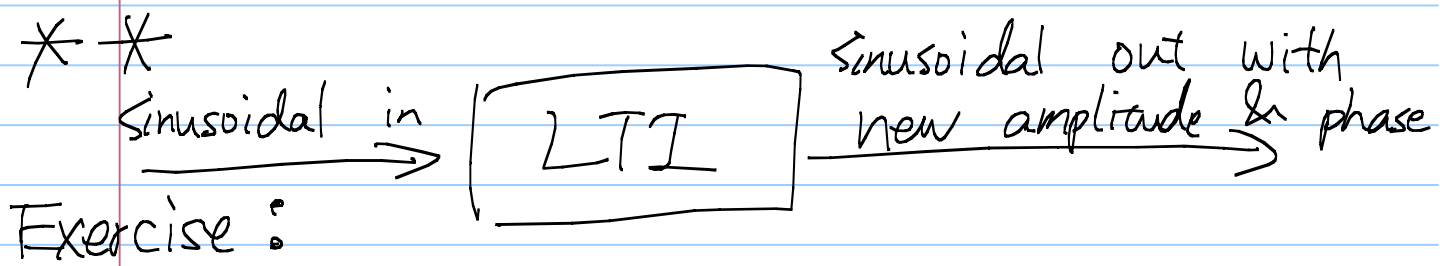
$$\Rightarrow \text{When } \omega=1, \quad x(t) = e^{j \cdot 1 \cdot t} = \cos(t) + j\sin(t)$$

$$y(t) = e^{jt} \left(\frac{1-j}{2} \right)$$

$$= e^{jt} \cdot \left(\frac{1}{\sqrt{2}} e^{j\left(\frac{\pi}{4}\right)} \right)$$

$$= \frac{1}{\sqrt{2}} \cos\left(t - \frac{\pi}{4}\right) + \frac{1}{\sqrt{2}} j \sin\left(t - \frac{\pi}{4}\right)$$

⇒ When the input is $\cos(t)$ ($\text{Re}(x(t))$)
 the output is $\frac{1}{\sqrt{2}} \cos\left(t - \frac{\pi}{4}\right)$ ($\text{Re}(y(t))$)



Q: Find out the output $y_2(t)$
 when the input is $x_2(t) = \cos(\sqrt{3}t)$

A: $y_2(t) = \frac{1}{2} \cos\left(\sqrt{3}t - \frac{\pi}{3}\right)$

∴ $\cos(t)$ → $\frac{1}{\sqrt{2}} \cos\left(t - \frac{\pi}{4}\right)$ $\cos(\sqrt{3}t)$ → $\frac{1}{2} \cos\left(\sqrt{3}t - \frac{\pi}{3}\right)$ ⇒ Low freq part is stronger, You feel stronger bass in the music

* Watch Video 2.3, 10