③ Associativity (serial concatenation)

$$x * (h_1 * h_2) = (x * h_1) * h_2$$

Question for the teams

pf 1: by integration

pf 2: by system construction.

in a similar way of proving distributivity.

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* The essence of an LTI system:

its impulse response $h(t)$ or $h[n]$

* We have learned

Given a system $\longrightarrow$ decide its classification

We will now learn

Given a LTI system $\longrightarrow$ decide its classification

$\downarrow$

find its impulse response

Classification #1: Memoryless (depends only on the present)

An LTI sys is memoryless if

$h[n] = K\delta[n]$ or $h(t) = K\delta(t)$

The reason is

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

commutativity.

$$= \sum_{k=-\infty}^{\infty} x[n-k]\,h[k]$$

$$= \cdots + \underbrace{x[n-1]h[1]}_{\text{past}} + \underbrace{x[n]h[0]}_{\text{present}} + \underbrace{x[n+1]h[-1]}_{\text{future}}$$

Memoryless $\Longleftrightarrow$ $h[k] = 0$ for $k \neq 0$

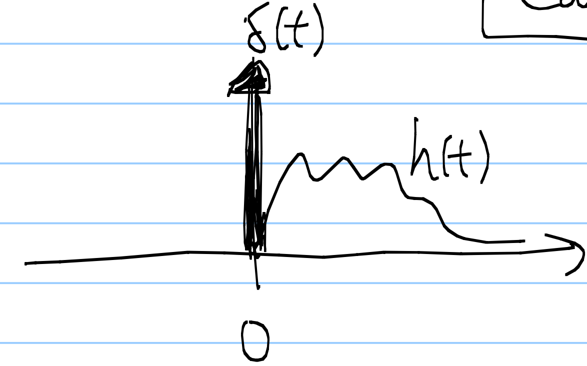
Classification #2: Causality (depending on the past & the present)

An LTI system is causal if

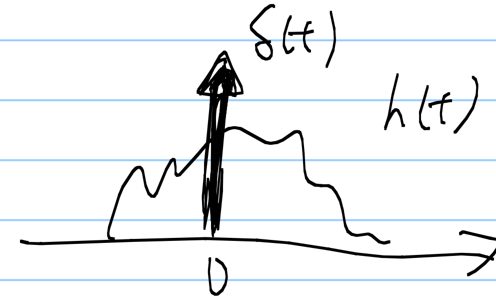
$h[n] = 0$ for $n < 0$ or $h(t) = 0$ for $t < 0$

Intuition

$\boxed{\text{Causal}}$



$\boxed{\text{Non-causal}}$



Causal

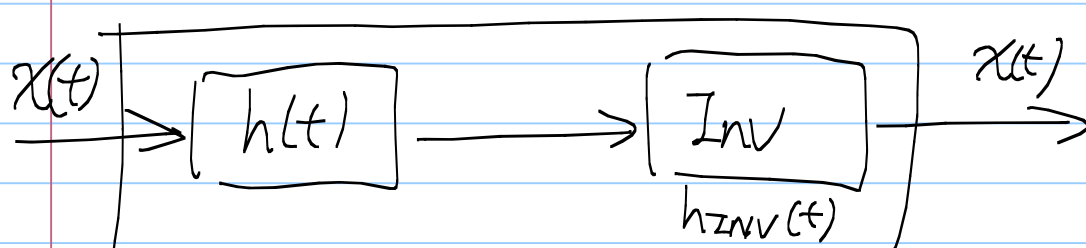
$\Leftrightarrow$ The output depends only on the present & the past.

$\Leftrightarrow$ The input $\delta(t)$ affects only the present & the future

$h(t) \neq 0$ only for $t \geq 0$

$h(t)$ should not anticipate the $\delta(t)$

* Classification #3: Invertibility
(whether there exists an inverse system.)



Q: Suppose the inverse sys has impulse response $h_{INV}(t)$. What is $h(t) * h_{INV}(t)$

Ans: If $\delta(t)$ is the input, the output must be $\delta(t)$.

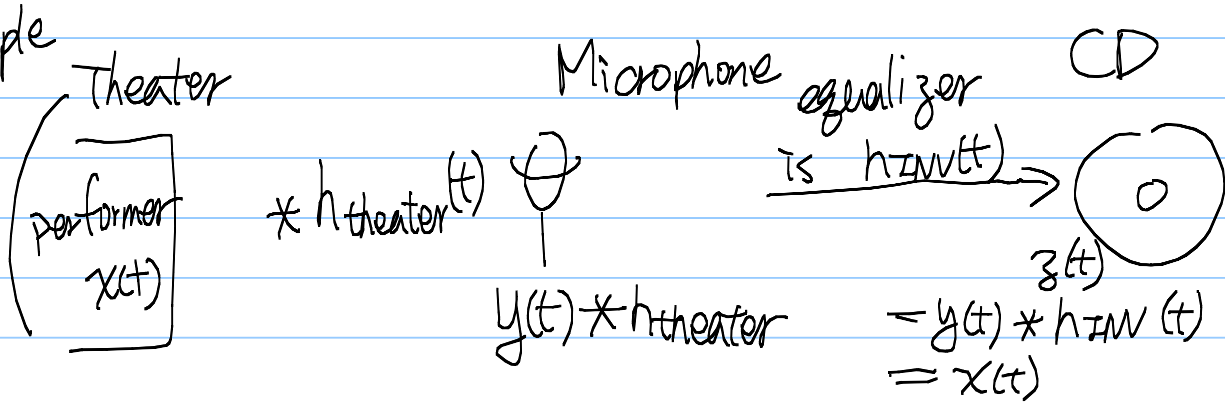
$\Rightarrow$ A LTI sys is invertible if there exists an $h_{INV}(t)$ (or $h_{INV}[n]$) such that $h(t) * h_{INV}(t) = \delta(t)$

Q: For any given $h(t)$, how to construct $h_{INV}(t)$ such that $h(t) * h_{INV}(t) = \delta(t)$?

Ans: We will answer this in the second half of the semester

Q: Why is it important to find $h_{INV}(t)$?

Example



Classification #4: Stability

(bounded input $\Rightarrow$ bounded output)

A LTI system is stable if

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \quad \text{or} \quad \int_{t=-\infty}^{\infty} |h(t)|\,dt < \infty$$

(That is a single impulse should not generate too much output. The output should die down after a while.)

Q: Classify the following system

	Memoryless	Causal	Stable
$h_1[n] = \left(\frac{1}{2}\right)^n$	W.M	NC	NS
$h_2[n] = \left(\frac{1}{2}\right)^n u[n]$	W.M	C	S
$h_3(t) = e^{-\|t\|}$	W.M	NC	S
$h_4(t) = e^{-t} u(t)$	W.M	C	S