Convolutional sum/
Convolutional integral

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k], \quad y(t) = \int_{-\infty}^{\infty} x(s)\,h(t-s)\,ds$$

mathematically.
They are just a sum & an integral

Intuitively.
Compute the output $y(t)$ from the input $x(t)$ & the impulse response $h(t)$

* Properties of convolution / LTI systems
(All properties apply both to the convolutional sums (DT) & convolutional integral.)

We use $x * h$ as short hand.

① Commutativity

$$x * h = h * x$$

pf: $x * h = \int_{-\infty}^{\infty} x(s)\, h(t-s)\, ds$. equal.

$$h * x = \int_{-\infty}^{\infty} h(s)\, x(t-s)\, ds$$

Choose $s' = t - s$, $ds' = -ds$

$s = t - s'$, $-ds' = ds$

$$= \int_{+\infty}^{-\infty} h(t-s')\, x(s')\, (-ds')$$

$x(t) \to \boxed{h(t)} \xrightarrow{y(t)}$ $= \int_{-\infty}^{\infty} h(t-s')\, x(s')\, ds$

$h(t) \to \boxed{x(t)} \xrightarrow{y(t)}$

② Distributivity

$$x * (h_1 + h_2) = x * h_1 + x * h_2$$

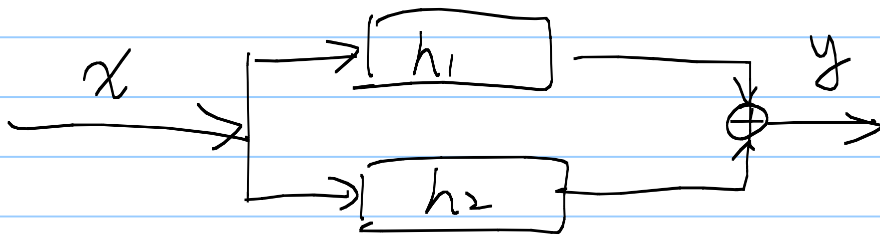
pf 1: By integration

$$\text{LHS} = \int_{-\infty}^{\infty} x(s)\, \big(h_1(t-s) + h_2(t-s)\big)\, ds$$

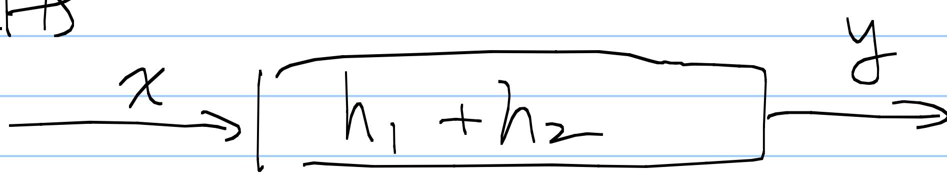
$$= \int_{-\infty}^{\infty} x(s)\, h_1(t-s)\, ds + \int_{-\infty}^{\infty} x(s)\, h_2(t-s)\, ds$$

$= \text{RHS}$.

pf 2: RHS:

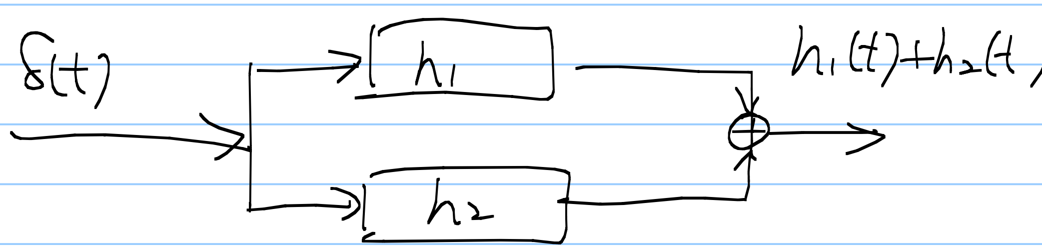


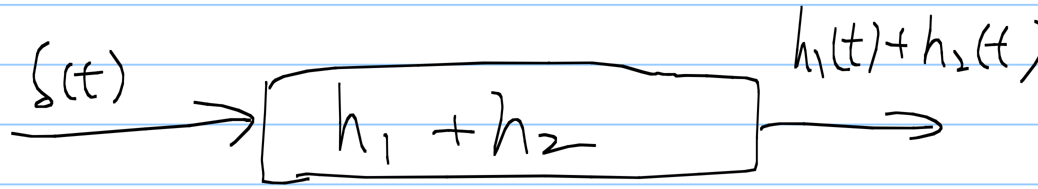
LHS



Q: Are the two systems identical

Ans: Since the impulse response is the "essence" of the system, we only need to check whether their impulse responses are identical





③ Associativity (serial concatenation)

$$x * (h_1 * h_2) = (x * h_1) * h_2$$

Question for the teams

pf 1: by integration

pf 2: by system construction.

in a similar way of proving distributivity.

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* The essence of an LTI system:

its impulse response $h(t)$ or $h[n]$

* We have learned

Given a system ⟶ decide its classification

We will now learn

Given a LTI system ⟶ decide its classification

$\downarrow$ $\nearrow$

find its impulse response