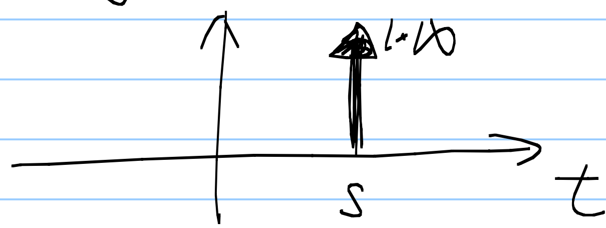
Note Title 2/3/2010

CT LTI Sys.

The test signals are $\delta(t-s)$: shifted impulses.



Recall

$$x(t) = \int_{-\infty}^{\infty} \underline{x(s)}\ \underline{\delta(t-s)}\, ds$$

$x(s)$ → coeff α_s

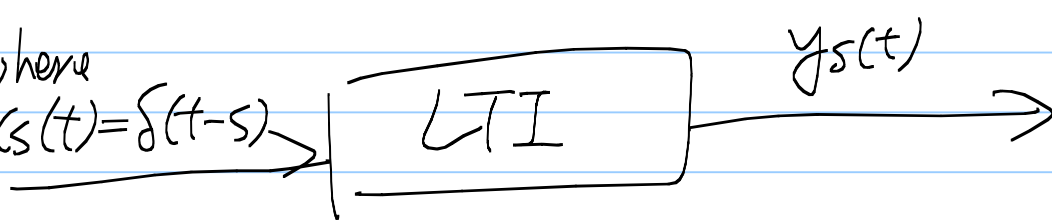
$\delta(t-s)$ → test signals $x_s(t)$

by linearity

$$y(t) = \int_{-\infty}^{\infty} \alpha_s\, y_s(t)\, ds$$

$$= \int_{-\infty}^{\infty} x(s)\, y_s(t)\, ds$$

where $x_s(t) = \delta(t-s)$

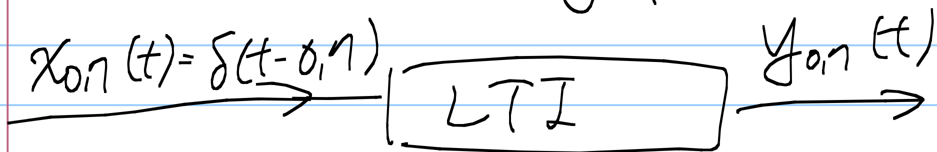


$y_s(t)$ is the output when the input is the shifted impulse $x_s(t) = \delta(t-s)$

Again we use $h(t)$ to denote $y_0(t)$, the output when the input is $\delta(t)$.

* $h(t)$ is thus termed the (unit) <u>impulse response</u>.

* Q: How about $y_{0,n}(t)$?



by Time-Invariance

$$y_{0,n}(t) = h(t-0,n)$$

$$\Rightarrow y(t) = \int_{-\infty}^{\infty} x(s)\, h(t-s)\, ds$$

$x(s)$ → coeff

$h(t-s)$ → the output of shifted impulses

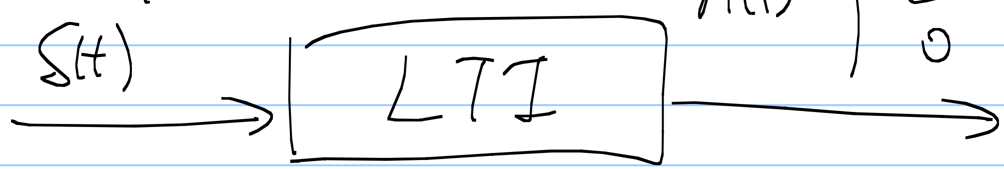
≡ the shifted "impulse response"

is denoted by

$y(t) = x(t) * h(t)$: the convolution integral of $x(t)$ & $h(t)$.

Theorem: For a CT LTI sys. with <u>impulse response</u> $h(t)$, the input/output relationship is $y(t) = x(t) * h(t)$

Ex: For a given LTI sys, and I know



$$h(t) = \begin{cases} e^{Tct} & \text{if } t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Q: What is the output $y(t)$ when the input is $x(t) = \begin{cases} 1 & \text{if } -3 \leq t < 0 \\ 0 & \text{otherwise} \end{cases}$?

Ans: $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(s)\, h(t-s)\, ds.$$

This is exactly HW3 Q25

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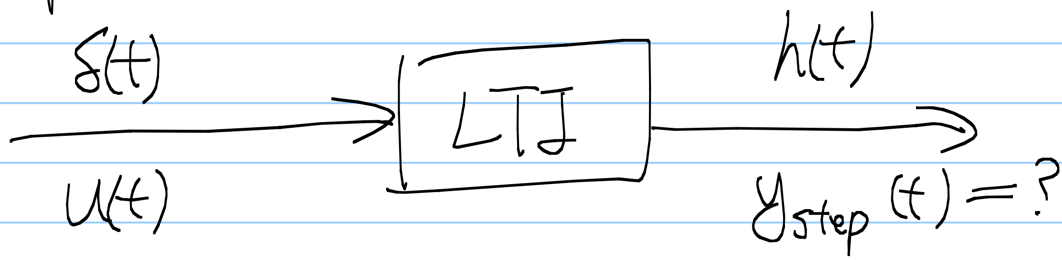
* Another example: Compute the **step response**.

Q: For a given LTI system with impulse response $h(t)$



Find out the step response $y_{step}(t)$.

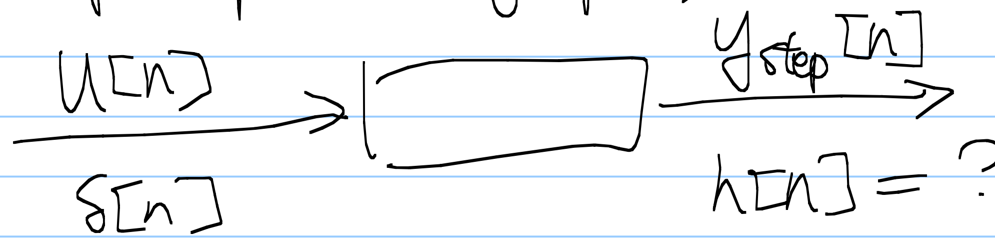
Ans: $y_{step}(t) = \int_{-\infty}^{\infty} u(s)\, h(t-s)\, ds$

↓ commutativity

$$= \int_{-\infty}^{\infty} h(s)\, u(t-s)\, ds$$

$$= \int_{-\infty}^{t} h(s)\, ds$$

Q: For a given LTI system with step response $y_{step}[n]$



Find out the impulse response.

Ans: $\delta[n] = u[n] - u[n-1]$

$\Rightarrow h[n] = y_{step}[n] - y_{step}[n-1]$ #